Benchmarking LGD Discount Rates

Harald Scheule[1], Stephan Jortzik[2]

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Abstract

This paper provides a theoretical and empirical analysis of alternative discount rate concepts for computing LGDs using historical bank workout data. It benchmarks five discount rate concepts for workout recovery cash flows to derive observed Loss rates Given Default (LGDs) in terms of economic robustness and empirical implications: contract rate at origination, loan weighted average cost of capital, return on equity, market return on defaulted debt, and market equilibrium return. The paper develops guiding principles for LGD discount rates and argues that the Weighted Average Cost of Capital (WACC) and market equilibrium return dominate the popular contract rate method. The empirical analysis of data provided by Global Credit Data (GCD) shows that declining risk-free rates are in part offset by increasing market risk premiums. Common empirical discount rates are between the risk-free rate and the return on equity. The variation of empirical LGDs is moderate for the various discount rate approaches. Furthermore, a simple correction technique for resolution bias is developed and increases observed LGDs for all periods, particularly recent periods.

Keywords: Default, Discount rates, Global Credit Data, LGD, Recovery, Resolution, Systematic risk

[1] UTS Business School, University of Technology, Sydney, PO Box 123, Broadway, NSW 2007, Australia; email: harald.scheule@uts.edu.au

[2] Stephan Jortzik, Chair of Global Credit Data Methodology Committee, www.globalcreditdata.org , email: Stephan.Jortzik@globalcreditdata.org

1 Introduction

The Loss rate Given Default (LGD) is a central modelling parameter in bank-internal credit risk models for credit risk exposures that measures the degree of shortfall of net recoveries relative to the outstanding loan amount due to and attributable to the default event. Credit risk exposures may result from assets, derivatives, credit lines and guarantees.

LGD modelling comprises of two stages. In the first stage, observed LGDs are inferred from the relative difference of the outstanding loan amounts and the sums of discounted observed recovery cash flows at default. In the second stage, observed LGDs are explained by risk factors that are observed prior to default. Here, a large number of econometric techniques including linear and non-linear, fractional response, beta regressions and more advanced machine learning techniques have been proposed (see Loterman et al., 2012, Baesens et al., 2016 and Roesch & Scheule, 2020 for details).[3]

LGD discount rates are therefore a key input parameter for the calculation of observed LGDs in the first stage. The methodology for LGD discount rates is controversial for a number of reasons. First, much of the extant literature on discount rates focuses on unconditional asset returns. Recovery cash flows are observed post-default and this conditionality has only been reflected in very few papers. Second, recovery cash flows are observed during a resolution time of multiple periods of different economic states and this mixing effect results in a limited linkage between bank loan LGDs and economic cycles (compare Qi & Yang, 2009 and Yao et al., 2017). As a result, links between systematic risk and discount rates have not been analysed. Third, most commercial banks are unable to observe market prices for recoveries. Some studies have analysed post default bond prices and find large discount rate ranges and hence uncertainty due to low observation counts.

This paper analyses five LGD discount rate concepts. First, the contract rate at origination (Contract) is the contractual interest rate of a loan as the combination of a base rate plus a risk premium at loan origination. Second, the loan weighted average cost of capital (WACC) as the bank funding costs is the combination of the capital ratio weighted cost of equity funding and the debt ratio weighted cost of debt funding. The capital ratio may be based on the loan-specific regulatory capital requirements. Third, the return on equity (ROE) is the cost of equity funding. Fourth, the market return on defaulted debt is the return on the bond price at resolution (i.e., the realised recovery at resolution) relative to the bond price at default (i.e., the expected recovery at default). Fifth, the market equilibrium return is the combination of a base rate plus a premium for systematic risk. The risk premium is based on the product of the systematic risk sensitivity and the equity risk premium.

[3] Note the concept of Stage I and Stage II modelling is common in the literature which analyses workout cash flows (e.g., Do et al., 2018). Different terms may exist. Some literature is based only on the (Stage II) modelling of observed LGDs (e.g., using one minus observed bond prices over par value at or shortly after default as a proxy for observed LGDs and does not require a two-stage approach. However, most LGD calculations performed by commercial banks do require both stages.)

These approaches may be separated into contract specific, comparable and equilibrium approaches and reflect a broad spectrum. The approaches analysed are based on a survey of GCD member banks for approaches that are currently implemented or have been considered during implementation and a review of the academic literature as well as guidance notes by regulators and accounting standards boards (see Section 3 for further details).

This paper provides the following **four specific contributions**. First, this paper develops guiding principles for LGD discount rates. Discount rates should be based on the opportunity costs of comparable financial instruments and include the risk-free rate and a premium for non-diversifiable risk. Discount rates should be based on information that is available at the time of default and exclude premiums for realised risk.

Second, the properties for five alternative discount rates are benchmarked in terms of these principles: simplicity, data availability and avoidance of negative LGD values. As a theoretical contribution, an extension to WACC is extended to a loan-specific WACC to be applicable to the data requirements of commercial banks.

Third, this paper tests the practical implementation of the approaches and implications on discount rates as well as LGDs using a large data set on historical workout data provided by the Global Credit Data (GCD).

Fourth, this paper provides a simple methodology for correcting for the resolution bias in LGDs. The bias is due to the fact that short resolution times result in low LGDs and most recent loss observations are dominated by short resolution times.

The paper finds that WACC and market equilibrium return dominate the popular contract rate method and this method has two main shortcomings. First, contract rates are based on the origination time: this is a violation of the principle that discount rates should relate to the default time. Second, contract rates include the expected loss in relation to default risk: this is a violation of the principle that discount rates should not compensate for realised risk. A trade-off effect is noted as WACC and market equilibrium return are model-based and are therefore somewhat more complicated to derive and validate.

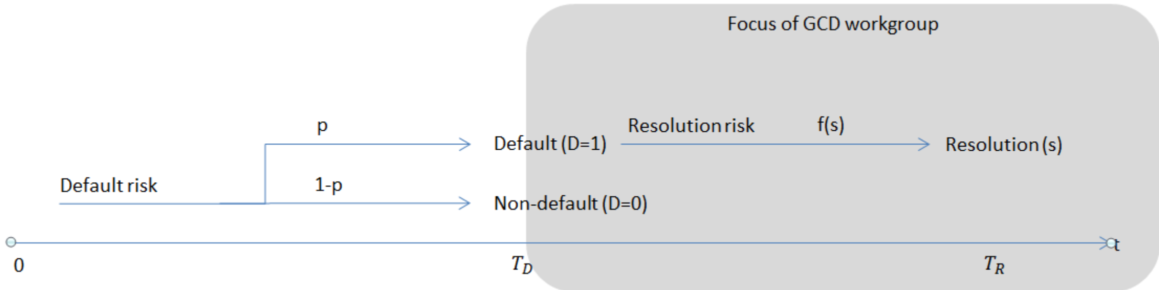
The empirical analysis shows that declining risk-free rates are in part offset by increasing market risk premiums in the period 2000-2013. This interaction implies limited variability of discount rates and observed LGDs in the empirical analysis. Common empirical discount rates are found to be between the risk-free rate and the return on equity. The variation of empirical LGDs is moderate for the various discount rate approaches. The correction for resolution bias increases observed LGDs for all periods. The dataset is provided by a consortium of large commercial banks and is one of the largest data sets for recovery cash flows available to researchers. GCD oversees the data collection process and ensures the homogeneity of the data collected over the various contributing banks. This paper summarises work that has been undertaken by a GCD work group on LGD discount rates.

The paper is structured as follows. Section 2 provides a Background on LGD modelling with regard to risk, expectation and realisation of LGDs. Section 3 develops guiding principles and analyses five different discount rate concepts. Section 4 provides the empirical analysis for GCD data. Section 5 concludes and provides an outlook for future works.

2 Background on LGD modelling: risk, expectation and realisation

Figure 1 shows the stylised evolution of credit losses from the beginning of the observation period (time 0) to the default event (time T_D), and to the resolution event (time T_R).[4] We consider two sequential risk processes: **default risk** with two possible outcomes (default and non-default), and **resolution risk** with an infinite number of possible outcome states S. These outcomes may include a random number of cash flows at random times between default time and resolution time. Examples for resolution categories are workouts, modifications, and cures.[5]

Figure 1: Evolution of credit losses



We follow the GCD terminology of "probability of default" being the forward estimate and "observed default frequency" or "observed default rate" being the observed. In this spirit, the WG has established a clear and simple terminology for LGDs, which is summarised in Table 1:

Table 1: LGD Terminology

Abbrev.	Description	Character	Discount	Application
LGD	LGD	Risk (random variable)	yes	Model framework
NLGD	Nominal LGD	Risk (random variable)	no	Model framework
ELGD	Expected LGD	Expectation	yes	Stage II LGD modelling
ENLGD	Expected Nominal LGD	Expectation	no	Avoid circular references
OLGD	Observed LGD	Realization	yes	Stage I LGD modelling
ONLGD	Observed Nominal LGD	Realization	no	Avoid circular references
DLGD[6]	Basel downturn LGD	Stress	n.a.	Regulatory capital

[4] Some reference values for the average time to resolution may be found in Araten et al. (2004): 0.8 years and Jacobs Jr (2012): 1.7 years. These values are consistent with the GCD experience.
[5] GCD has collected a large number of observations of default outcomes (resolutions).
[6] Downturn LGD is subject to regulatory definition and is not the focus of this study.

A number of different LGD definitions for our discount rates is required to solve circular references in the empirical execution and to derive regulatory capital for the WACC approach. This paper distinguishes between the random and hence unknown LGD, the expectation in an actuarial sense at the time of default and the realisations of LGDs do not consider discounting of cash flows and are applied to resolve circular references. Furthermore, Basel III is based on the concept of Downturn LGDs, which are expected LGDs during an economic downturn stage. This is generally derived from expected LGDs and does not require the application of a discount rate, which is why a "n.a." was assigned. Note, however, that the model input expected LGD is based on a discount rate.

Cures are often considered in Stage II LGD modelling (see, e.g., Do et al, 2018) and these are not separately analysed in the derivation of discount rates and assume a value of zero.

2.1 Default risk

Default risk is assumed to have two possible realisations $D \in \{0,1\}$.[7] The associated random loss rate given the random LGD at time of default is:

$$L_{T_D} = \begin{cases} 0, \text{ if D=0} \\ LGD, \text{ if D=1} \end{cases} \tag{1}$$

The variable LGD will be considered in detail in the next section. The probability distribution of the default indicator D is Bernoulli:

$$p(D) = \begin{cases} 1-p, \text{ if D=0} \\ p, \text{ if D=1} \end{cases} \tag{2}$$

The outcome of default risk is random (hence unknown) and the expected loss rate (EL) at default may be computed by summing up the probability weighted discrete loss outcomes:

$$EL_{T_D} = (1-p) * 0 + p * E\left(LGD_{T_D}\right) = p * E\left(LGD_{T_D}\right) \tag{3}$$

The present value at origination of future loan losses results from discounting the outcomes to the origination (time T_0):

$$EL_0 = (1-p) * 0 + p * E\left(\frac{LGD_{T_D}}{\left(1+rf_{[0,T_D]}+\delta_{[0,T_D]}\right)^{T_D-0}}\right) = \frac{p*E\left(LGD_{T_D}\right)}{\left(1+rf_{[0,T_D]}+\delta_{[0,T_D]}\right)^{T_D-0}}. \tag{4}$$

The discount time and discount rate are equal to the risk-free rate $rf_{[0,T_D]}$ and a risk premium $\delta_{[0,T_D]}$ for all resolution outcomes and can hence be written outside the expectation operator as all resolution outcomes relate to the same time of default.

The discount rate for default risk includes a risk premium that is based on the systematic risk of the default process and is (state) independent of the (unknown) outcome of default risk as all risk premiums

[7] Note that probabilities of default are considered in the contract rate concept.

are (compare Damodaran (2007) and the guiding principles for risk based discount rates in Section 3.2). The discount rate may be period-specific ($[0, T_D]$) to reflect the term structure of discount rates.

2.2 Resolution risk

Resolution risk is assumed to have an infinite amount of possible realisations. The random net recovery cash flows for a given realisation path *s* may occur any time between default time and the resolution time: $c(s) = \left(c_{t_1}(s), \dots, c_{T_R}(s)\right)$ and the probability density function is $f(s)$, with $\int f(s) = 1$.

The random absolute (e.g., in dollar terms) nominal LGD (ANLGD) is then:

$$ANLGD(s) = EAD - \sum_{t=T_D}^{T_R} c_t(s). \tag{5}$$

The random relative nominal LGD (NLGD) per scenario is computed by relating ANLGD to the exposure at default (EAD), which comprises the outstanding principal and accrued interest and is assumed to be deterministic, rather than an ex-ante bank estimate:[8]

$$NLGD(s) = \frac{ANLGD(s)}{EAD} = 1 - EAD^{-1} \sum_{t=T_D}^{T_R} c_t(s). \tag{6}$$

The random LGD at default time follows by the means of discounting by a risk-adjusted rate:

$$LGD(s) = 1 - EAD^{-1} \sum_{t=t_1(s)}^{T_R(s)} \frac{c_t(s)}{\left(1 + rf_{[T_D,t]} + \delta_{[T_D,t]}\right)^{t-T_D}}. \tag{7}$$

The discount rate is the same for all loss observations and (state) independent of the (unknown) outcome of resolution risk as all risk premiums are (compare the guiding principles for risk based discount rates in Section 3.2). The objective of the Working Group is the computation of observed LGDs based on the time value of money – here $rf_{[T_D,t]} + \delta_{[T_D,t]}$. The discount rate may be period-specific ($[T_D, t]$) to reflect the term structure of discount rates.

The outcome of resolution risk is random (hence unknown ex-ante) and the expected nominal LGD (ENLGD), which is unconditional on any risk factor, may be computed by integrating over the density weighted random *NLGD(s)*:

$$ENLGD = E(NLGD) = \int f(s) NLGD(s)\, ds. \tag{8}$$

Likewise, the present value of the expected LGD is:

$$ELGD = E(LGD) = \int f(s) LGD\,(s) ds. \tag{9}$$

[8] Corporate loans often have deterministic amortisation schedules rather than embedded pre-payment options and liquidity facilities.

2.3 Estimation of LGD

The observations of the random variables NLGD (LGD) may be called observed NLGD (LGD), realized NLGD (LGD), ultimate NLGD (LGD), or post resolution NLGD (LGD). We will call this observed nominal LGD (ONLGD) and observed LGD (OLGD), which can be estimated, based on Equations (6) and (7). This is often referred to as Stage I LGD modelling.

The observed LGDs may be linked with risk factors (via the estimation of regression models) and LGD may be estimated conditional on risk factors. The resulting estimated LGDs may be applied to compute bank-internal and regulatory (Basel III) capital requirements. This is often referred to as Stage II LGD modelling.

Furthermore, the expected NLGD (LGD) may be estimated based on assumptions. This paper applies the average over the ONLGDs (OLGDs) for given risk segments as the simplest approach.

3 Discount rates for computing empirical LGDs

3.1 Regulatory guidance

The Basel Committee on Banking Supervision (2005) mandates that the discount rate includes the time value of money and a risk premium for undiversifiable risk:

"[…] When recovery streams are uncertain and involve risk that cannot be diversified away, net present value calculations must reflect the time value of money and a risk premium appropriate to the **undiversifiable risk**. In establishing appropriate risk premiums for the estimation of LGDs consistent with economic downturn conditions, the bank should focus on the uncertainties in recovery cash flows associated with defaults that arise during the economic downturn conditions identified under Principle 1. When there is no uncertainty in recovery streams (e.g., recoveries derived from cash collateral), net present value calculations need only reflect the time value of money, and a risk-free discount rate is appropriate."

Variations of these regulations and guidance notes are included in national guidance notes from prudential regulators. Examples are Australia (Australian Prudential Regulation Authority, 2005), Hong Kong (Hong Kong Monetary Authority, 2006), the UK (Financial Services Authority, 2003, Bank of England, 2013), and the US (Office of the Comptroller of the Currency et al., 2003, 2007).

The Prudential Regulation Authority (PRA) of the Bank of England (see Bank of England, 2013) expects banks "[…] to ensure that no discount rate used to estimate LGD is less than 9%. The publication further found that "[…] there was no widely-accepted industry approach to determining appropriate discount rates, insufficient evidence of the appropriateness of rates; and a tendency to reduce discount rates over time […]". The PRA accepts lower discount rates if conservative cash flows (e.g., certainty equivalent cash flows) are applied. Furthermore, the PRA is willing to consider alternative industry approaches subject to review.

Most recently the European Banking Authority (EBA, 2017) has suggested a discount rate equal to the primary interbank offered rate applicable at the moment of default plus 5%.[9]

Accounting standards differ from this interpretation as they prescribe the use of the effective rate. The effective rate is the rate that exactly discounts expected future cash payments or receipts through the expected life of the financial instrument to the initial investment. This is generally the contract rate for fixed rate loans and the current interest rate for floating rate loans. US banks have been subject to Statement of Financial Accounting Standards No.114 (Accounting by Creditors for Impairment of a Loan, FAS-114) which prescribes the effective interest rate as discount rate. For international accounting standards, IAS 39 prescribes the effective interest rate of the financial assets. In the future, larger banks will be subject to International Financial Reporting Standards (IFRS) established by the International Accounting Standards Board (IASB). In essence, IFRS 9 continues to prescribe the discount rate to be the effective interest rate.

More recently, the International Financial Reporting Standard 9 (IASB, 2014) and Current Expected Credit Loss (CECL, FASB, 2016) require the computation of lifetime expected losses as a basis for loan loss provisioning. IFRS 9 has been implemented from 2018 onwards outside the US. The standard applies to all instruments measured at amortized cost and instruments measured at fair value through other comprehensive income but lifetime expected losses are only computed for assets that have increased significantly in credit risk. CECL is implemented in the US and applies to all assets.[10] There are a number of approaches to implement the new regulations and discounted cash flow methods are popular. FASB specifies: "If an entity estimates expected credit losses using methods that project future principal and interest cash flows, the entity shall discount expected cash flows at the financial assets' effective interest rate.[…]" Note that IFRS 9/CECL are a different application to the computation of observed LGDs, as IFRS 9/CECL generally require a pre-default calculations while LGDs generally require post default calculations. However, this is another example for the popularity of the contract rate/effective interest rate.

In summary, prudential regulators are open to various approaches for LGD discount rates on economic substance provided they include the time value of money and a premium for non-diversifiable (systematic) risk. Contrary to this, accounting standards prescribe the use of the effective rate (i.e., the contract rate in most instances).

9 The approaches suggested by the EBA and the PRA are not analysed further as these do not take the individual loan and bank characteristics into account.

10 For further details see Bellini (2019).

3.2 Guiding principles for LGD discount rates

We focus in this paper on discounted cash flow (DCF) methods as this matches the data environment in the banking industry. For example, GCD collects observed cash flows during work out on mostly non-traded instruments. In this technique, expected cash flows are discounted by risk adjusted discount rates. Observed LGDs are computed by discounting observed cash flows by the risk adjusted discount rate. The WG proposes the following guiding principles:

- Discount rates should be based on the opportunity costs of financial instruments with comparable price-relevant factors to avoid arbitrage opportunities for non-regulated market participants and hence, market inefficiencies.
- Key price factors per regulatory guidance includes the risk-free rate and premium for non-diversifiable risk. We interpret non-diversifiable risk as systematic risk that cannot be diversified in financial markets.
- Discount rates should be based on information that is available at the time of default. This includes parameter estimates based on historic information.
- Discount rates should exclude premiums for realised risk. Discount rates should reflect resolution risk only and thus, (i) not include a premium for default risk, and (ii) not exclude the premium for resolution risk.

This paper does not consider, but encourages a more detailed analysis of the following aspects:

- Recoveries may be decomposed into assets (determined ex-ante) of different degrees of systematic risk. These assets may be appraised using discount rates that reflect the difference in systematic risk levels.
- Banks may apply conservative estimates for recovery realisations and a certainty equivalent method with the risk-free rate as discount rate, as appropriate.
- Basel Committee on Banking Supervision (2005) mandates that "[…] appropriate risk premiums for the estimation of LGDs consistent with economic downturn conditions […]" are applied. There are a number of approaches available to model Downturn LGD. EBA (2019) distinguished three general categories: first, model building approaches for banks which have sufficient loss data for the identified downturn period, second, haircut or extrapolation approaches for banks which do not have sufficient loss data, and third, an addition of 15 percentage points to ELGD estimates if banks cannot use approaches in the first two categories. Banks should carefully consider any interaction between the (i) discount rate method, (ii) the downturn method and the data on which (i) and (ii) are based.
- Impact of taxation on cash flows and discount rates.
- Term structure of interest rates.

3.3 Review of discount rate approaches

The approaches analysed in the following are based on a survey of GCD member banks and are currently implemented or have been considered during implementation and a review of the academic literature, as well as regulatory guidance notes by regulators and accounting standards boards (see Section 3.1 for further details). The approaches may be separated into loan contract specific, comparable and equilibrium approaches.

3.3.1 Loan contract rate

Asarnow & Edwards (1995) propose the loan contract rate of the defaulted loan as discount rate.

Default and resolution risk are priced in the contract rate. The annual **return given a non-default** is the contract rate k

$$\begin{aligned} r_{D=0} &= (1+k)-1 \\ &= k \end{aligned}, \tag{10}$$

With reference to Figure 1, we focus on default risk and set the outcome after default to ELGD. We assume a loan with a maturity of one year and default at maturity. Assuming that interest (based on the contract rate) has accrued to a full period, the annual **return given a default** is

$$\begin{aligned} r_{D=1} &= (1+k)(1-ELGD)-1 \\ &= k(1-ELGD)-ELGD \end{aligned}. \tag{11}$$

Therefore, the **expected return** with regard to default risk is:

$$\begin{aligned} E(R_L) &= (1-p)r_{D=0} + pr_{D=1} \\ &= (1-p)k + p(k(1-ELGD)-ELGD)' \end{aligned} \tag{12}$$

with the probability of default p and the random return on the loan (index L) R_L. Note that the expected return for resolution risk is embedded in ELGD in the above equations.

In contrast to the expected return with regard to resolution risk, the **contract rate** at origination includes the expected loss, and the LGD-adjusted likelihood of non-default (see also Chalupka & Kopecsni, 2008). This becomes apparent after solving Equation (12) for *k*:

$$k = \left(E(R_L) + \underbrace{pELGD}_{\text{expected } Loss}\right) \Big/ (1-pELGD). \tag{13}$$

The contract rate has two shortcomings: (i) it relates to the expected return with regard to default risk and resolution risk, which is set at origination,[11] and (ii) it exceeds the expected return for the loan as it

[11] The systematic risk may change post default with the realisation of default risk. The workgroup did not discuss this in more detail but encourages further research.

includes the expected loss. As a result, the contract rate contradicts the guiding principles presented in Section 3.2. Alternatively, the expected return implied in the contract rate from Equation (12) may be used as this is more closely aligned with the guiding principles.

However, estimated expected returns imply a circular reference to estimated ELGD assumption, which require the knowledge of the discount rate. To avoid the circular reference, one may apply ENLGD for the realised return (p.a.) given a default:

$$r_{D=1} = \left((1+k)(1-ENLGD)\right)^{1/((T_D-0)+(T_R-T_D))} - 1, \tag{14}$$

The formula assumes that all cash flows have been received at the time of resolution and computes the internal rate from the resolution time to the loan origination.

A length of one period may be assumed for the time of loan origination to default $(T_D - 0)$ for simplicity. The resulting expected return with regard to default risk and resolution risk is:

$$\begin{aligned} E(R_L) &= (1-p)r_{D=0} + pr_{D=1} \\ &= (1-p)k + p\left(\left((1+k)(1-ENLGD)\right)^{1/(1+T_R-T_D)} - 1\right)' \end{aligned} \tag{15}$$

R_L is the return on the loan, which is based on the contract rate and hence, only indirectly on financial markets. Furthermore, Office of the Comptroller of the Currency et al. (2003)[12] propose the contractual rate of the highest risk grade. This proposal is very similar to using rates which may apply after covenant violations, or alternatively, the contract rate at default. Note that contract rates for high idiosyncratic risk exposures and prior or at default are likely to include the pre-default expected loss and a similar correction may apply.

Discount rate example: the estimated probability of default is 1%, the contract rate is 5%, ENLGD is 60%, and time to resolution is 2 years. Therefore, the estimated expected return (and hence discount rate) with regard to default risk is 4.7% following Equation (15*)*

$$E(R_L) = (1-0.01)0.05 + 0.01\left(\left((1+0.05)(1-0.6)\right)^{1/(1+2)} - 1\right) = 0.0470.$$

3.3.2 Bank weighted average cost of capital (WACC)

Witzany (2009)[13] and Jensen (2015) have proposed the application of the weighted cost of capital as a

[12] The Office of the Comptroller of the Currency et al. (2003) stipulate that "A bank must establish a discount rate that reflects the time value of money and the opportunity cost of funds to apply to recoveries and costs. The discount rate must be no less than the contract interest rate on new originations of a type similar to the transaction in question, for the lowest-quality grade, in which a bank originates such transactions. Where possible, the rate should reflect the fixed rate on newly originated exposures with term corresponding to the average resolution period of defaulting assets." This approach is no longer found in the updated guidance Office of the Comptroller of the Currency et al. (2007).

[13] Witzany (2009) initially presents a CAPM Model, which results in a WACC model for the discount rate and computes (iteratively) the spread as the capital risk charge times the regulatory capital for market risk.

discount rate. Weights are generally based on the relative proportion of equity and debt funding of a bank in market value terms.

It is often argued that the bank funding costs do not reflect the risk profile of individual credit exposures with regard to resolution risk given loan default (compare e.g., Chalupka & Kopecsni, 2008). Furthermore, market funding costs for distressed/defaulted assets would be the preferred approach but very difficult to determine. Therefore, the WG has explored the computation of loan-specific capital and debt ratios based on the capital requirements of defaulted loans relative to the post-default expected loan value. Under the assumption that regulatory capital is a reasonable measure for systematic risk and that post-default capital and debt ratios are consistent with the ones of other (e.g., pre default) loan instruments, bank funding costs may provide a basis for a reasonable discount rate.[14]

Default risk is realised with the default event. Hence, bank capital $\mathrm{E}_{\mathrm{T_D}}$ is recomputed using a PD of unity (Equation 16) and the post-default loan value $\mathrm{V}_{\mathrm{T_D}}$ is assumed to equal the expected recovery (Equation 17):

$$\mathrm{E}_{\mathrm{T_D}} = (\mathrm{DLGD} - \mathrm{ELGD})\mathrm{EAD} \tag{16}$$

$$\mathrm{V}_{\mathrm{T_D}} = (1 - \mathrm{ELGD})\mathrm{EAD} \tag{17}$$

Hence, the post-default capital ratio (i.e., weight) e_{T_D} is:

$$e_{T_D} = \frac{E_{T_D}}{V_{T_D}} = \frac{DLGD - ELGD}{1 - ELGD}, \tag{18}$$

With the post-default equity value assumed to be equal to the unexpected loss given default:

$$\mathrm{E}_{\mathrm{T_D}} = DLGD - ELGD \tag{19}$$

The debt ratio (i.e., weight) m_{T_D} follows as $1 - e_{T_D}$.

We use index C for the expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk. This expected return may be based on the combination of the expected return on equity weighted by the post-default capital ratio and the expected return on debt weighted by the post-default debt ratio:

$$E(R_c) = e_{T_D}E(R_E) + m_{T_D}\,E(R_M) = \tag{20}$$

$$= \frac{DLGD - ELGD}{1 - ELGD}\,E(R_E) + \left(1 - \frac{DLGD - ELGD}{1 - ELGD}\right)E(R_M)$$

Structurally, this approach combines bank-level funding costs and loan-level funding ratios: e_{T_D} (and therefore m_{T_D}) are loan specific as DLGD and ELGD are based on Stage II LGD models while $E(R_E)$ is

[14] Prudential regulators have increased the focus on consistency between the various regulations (see Basel Committee on Banking Supervision, 2014).

the cost of equity (e.g., ROE) and $E(R_M)$ is the cost of debt (e.g., average bank debt funding costs). In other words, the equity and debt weights are loan-specific while the funding costs are bank-specific. In practice, $E(R_E)$ might be based on a Capital Asset Pricing Model or an extension thereof using the bank beta and market premium (see below for further details) while $E(R_M)$ may be derived from the weighted average cost of bank liabilities (in particular deposit rates, and wholesale funding costs).

In practice, the expected return on equity and debt may be approximated by equity and debt funding costs. The formula implies a circularity issue as ELGD and DLGD are required which in turn are in practice estimated from OLGD, which requires the discount rate.

Discount rate example: the estimated probability of default is 1%, estimated ELGD is 60%, DLGD is 63.2%, the post-default capital ratio is (63.2%-60%)/(1-0.6)=8%, the debt funding costs are 4% and the return on equity is 7.8%. Therefore, the estimated expected return (and hence discount rate) with regard to resolution risk is 4.30%:

$$E(R_c) = 0.08 * 0.078 + 0.92 * 0.04 = 0.0430$$

3.3.3 Bank return on equity

Eales & Bosworth (1998) discuss a range of discount rate approaches and decide to apply the return on equity as the discount rate. Their justification is that equity is debited/credited for differences between ELGD and OLGD and that the risk exposure of LGDs corresponds to the equity position. The approach may be interpreted as a special case (i.e., upper boundary) of the WACC approach where the capital ratio is 100%:

$$E(R_c) = E(R_E) \tag{21}$$

We use index C for the expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk. For example, the cost of equity may be computed using the capital asset pricing model:

$$E(R_E) = rf + \beta\, E(R_M - rf), \tag{22}$$

with the expected market excess return $E(R_M - rf)$ and $\beta = \frac{Cov(R_E,R_M)}{Var(R_M)}$. R_M is the return of the market portfolio which is somewhat controversial in the literature but for practical matters it is often replaced by the return of the local share market index and rf is the risk-free rate. Averages for realised market excess returns may be taken as proxies for expected values and are reported for broadly based equity market indices in various geographies.[15] Other approaches such as using average share returns (after adjusting for dividend payments), bank ROE target numbers, or accounting ROE ratio may be

15 Depending on historic time period and geography, market risk premiums are between 2% and 8% p.a., see Dimson et al. (2011).

considered.

Discount rate example: the bank equity beta to the market return is 0.8, the risk-free rate is 3% and the estimated expected market excess return is 6%. Therefore, the estimated expected return on equity (and hence discount rate) is 7.8%:

$$E(R_c) = 0.03 + 0.8 * 0.06 = 0.078.$$

3.3.4 Market return of marketable credit instrument

Brady et al. (2006) and Jacobs Jr (2012) compute the realised returns and average realised returns for defaulted bonds. In a similar fashion, returns on distressed debt (see Altman & Kuehne, 2012, who look more broadly at returns of distressed bonds) may be applied. The realised return (p.a.) given a resolution outcome *b* of a marketable financial instrument is:

$$r_{b,c} = \left(\frac{B(T_R)}{B(T_D)}\right)^{1/(T_R - T_D)} - 1, \tag{23}$$

with the instrument price B. We use index c for the return for an instrument that is comparable to a defaulted exposure subject to resolution risk. $B(T_R)$ is the bond price at resolution time T_R and $B(T_D)$ is the bond price at default time T_D. $T_R - T_D$ is the resolution (i.e., workout) period. Note that it is common to use the bond price 30-45 days past default as a proxy for the bond price at default. A positive (negative) return results if bond prices at resolution time exceed (are below) bond prices at default time. Especially short resolution periods, in combination with bond price changes from default time to resolution time, and result in large returns and large differences of mean returns for different sub-samples.

The expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk may be based on the average as an estimate for the expected return:

$$\begin{aligned} E(R_c) &= \frac{1}{B}\sum\nolimits_{b=1}^{B} r_{b,c} \\ &= \frac{1}{B}\sum\nolimits_{b=1}^{B}\left(\left(\frac{B_b(T_R)}{B_b(T_D)}\right)^{1/(T_{b,R} - T_{b,D})} - 1\right) \end{aligned} \tag{24}$$

For simplicity, we have assumed that the prices of all B defaulted bonds are observed and that the market expectations are based on the mean of realised returns.

Discount rate example: the bond price at resolution for two defaulted bonds is 55 and 45, the time to resolution is one and two years, the bond price at default is 40 for both bonds. The realised annual return for the first bond is (0.55/0.4)^(1/1)-1=0.375 and for the second bond (0.45/0.4)^(1/2)-1=0.0607. Therefore, the estimated expected return (and hence discount rate) with regard to resolution risk is 21.78%:

$$E(R_c) = 1/2(0.375 + 0.0607) = 0.2178$$

3.3.5 Market equilibrium return

Maclachlan (2004) suggests basing discount rates on market equilibrium models, such as the capital asset pricing model (CAPM). In these models, the expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk may be based on the expected return of an asset, which in turn is based on the risk-free rate, the beta and the market risk premium (i.e., the return of the market in excess of the risk-free rate):

$$E(R_E) = rf + \beta\, MP, \tag{25}$$

with the risk-free rate rf, the beta $\beta = \frac{Cov(R_C,R_M)}{Var(R_M)} = \frac{\mathrm{Corr}(R_C,R_M)\mathrm{Std}(\mathrm{C})}{\mathrm{Std}(R_M)}$, and the market risk premium $E(R_M - rf)$. cov(.) is the covariance, var(.) is the variance, Corr(.) is the correlation, and SD(.) is the standard deviation. We use index E as the expected return is based on an equilibrium model. The CAPM requires a number of assumptions such as frictionless financial markets that are controversial during economic downturns when default events are most likely to occur. Examples include the ability of market participants to borrow and lend, the absence of transaction costs and availability of all information to all market participants.

Discount rate example: the defaulted debt beta with regard to the market return is 0.5, the risk-free rate is 3% and the estimated expected market excess return is 6%. Therefore, the estimated expected return (and hence discount rate) with regard to resolution risk is 6%:

$$E(R_c) = 0.03 + 0.5 * 0.06 = 0.06.$$

Maclachlan (2004) estimates the defaulted debt beta based on:

- Correlation between the return on a defaulted bond index or defaulted bonds and the market return. The analysis results in defaulted debt betas of 36% based on the NYU Bond Index and 37.1% based on 90 defaulted US bonds. The market index was the S&P 500 index in both analyses.
- Asset correlations of 17% from Frye (2000),[16] resulting in defaulted debt betas of 30% based on an asset correlation.

The beta coefficient may be computed as follows

$$\beta_{i,j} = \frac{\sigma_{i,j}\sqrt{AC_j}}{\sigma_{M,j}} \tag{26}$$

[16] Note that Basel III provides asset correlations between 12% and 24% for corporate credit exposures in the Internal Ratings-based Approach.

with borrower i and risk segment j .

Alternative approaches to determine $\beta_{i,j}$ from Equation (26) include distressed bond and equity prices of defaulted borrowers. However, such approaches are limited by the number of observations and the separation of systematic and idiosyncratic risk. The WG believes that further research should be undertaken. The approaches for the risk premium suggested by the work group was to regress recovery rates on equity market excess returns or Fama-French factors.

Bank loan recoveries are expected to be correlated with GDP, which in turn is highly correlated with consumer consumption and the Consumption CAPM may provide for a price link.[17] Roesch & Scheule (2012) model empirical asset correlations AC_j by conditioning on historic average LGDs (e.g., through the cycle models) and time-varying information (e.g., point-in-time models).[18] Assuming a normal distribution, the natural logarithms of observed recovery rates at a certain point in time (ORR_{ijt}=1-$OLGD_{ijt}$) may be described as follows:

$$ln\left(ORR_{ijt}\right) = \alpha_j + \beta_j x_{jt} + \gamma_j \varepsilon_{jt} + \delta_j \varepsilon_{ijt,} \tag{27}$$

with borrower i, risk segment j and default time t. The parameters may be estimated risk segment-specific. α_j is an intercept, β_j the sensitivity to observable (in particular macro-economic) information, γ_j the sensitivity to a standard normally distributed systematic random effect and δ_j the sensitivity to an idiosyncratic standard normally distributed error term. γ_j and δ_j may also be interpreted as standard deviations. x_{jt} are time-varying systematic variables (here the lagged average log recovery and GDP growth). Other variables including loan-specific variables are possible. The model may be extended to control for selection issues by the consideration of default events and cure events (see Roesch & Scheule, 2012).

The asset correlation (AC) follows from Equation (27):

$$AC_j = \frac{\gamma_j^2}{\gamma_j^2 + \delta_j^2} \tag{28}$$

3.4 Benchmarking of discount rate approaches

The WG has discussed these concepts in light of a number of criteria:

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17 Note that Qi & Yang, 2009 and Yao et al., 2017 show that LGD variation is mainly explained by the collection policy, i.e., idiosyncratic characteristics and that there are limited linkages to the economic cycles. Another factor may be the variation of economic states over the resolution period during which recovery cash flows are collected.

18 This is comparable to default risk modelling, where asset correlations have been measured by linking time-varying defaults and default rates to historic average default rates (e.g., TTC models). In extensions, frailty effects conditional on observable time-varying information (e.g., PIT models) have been estimated (compare, e.g., Roesch & Scheule, 2020.

- Guiding principles of discount rates: degree to which an approach is supported by the principles for discount rates established by the WG. WACC and equilibrium returns meet these properties, while the other approaches violate aspects of these principles:
    - Contract rate: the concept includes a compensation for default risk (in essence the credit spread), which has been realised post default. Furthermore, contract rates relate generally to the origination time, with regard to systematic risk and the time value of money;
    - Return on equity: the concept assumes that the systematic risk is equal to an equity investment in the bank, which is unlikely to be reasonable given the heterogeneity of defaulted loans. The bank-specific return on equity is unlikely to include an appropriate premium for the systematic resolution risk of the defaulted loan;
    - Market returns of defaulted bonds: the concept is based on a small number of bond defaults and resulting returns are unlikely to provide for robust comparable discount rates.
- Simplicity: the approach is based on available measurable information, which does not require assumptions in the measurement process:
    - Contract rates: are generally observable but may be unclear for credit lines, derivatives and guarantees;
    - WACC and return on equity: require a moderate level of assumptions;
    - Market returns of defaulted bonds: comparability of defaulted bonds and defaulted loan is unclear (in particular for SME loans);
    - Equilibrium returns: require strong assumptions for the estimation of measures for systematic risk and computation of risk premiums.
- Application to empirical data:
    - Contract rates: may not be observable for all loans or may be complicated by front-end, back-end, hybrid or variable features;
    - WACC and return on equity: data on bank funding costs may not be available. Data may support the computation of the loan-level capital and debt ratio for WACC;
    - Market returns on defaulted bonds: data on defaulted bonds may not be available;
    - Equilibrium returns: loss data may be used to estimate the exposure to systematic risk.
- Negative LGDs: the realised LGD is computed based on the book value of EAD (i.e., expected outstanding principal at default) and the market value of resulting post-default realised LGD if

the current interest rates and current spreads for systematic risk are applied. Low interest rate regimes (e.g., as a consequence of monetary easing) in conjunction with high net recovery cash flows (e.g., in the instance of a cure) may result in present values in excess of EAD and hence negative LGDs. For example, a default followed by a loan service according to schedule and discount rates (determined at default) below the contract rate (determined at origination) would result in a recovery rate greater than one and hence a negative LGD. LGD values that are constrained within the interval [0,1] may support general acceptance and the input requirements of some Stage II regression models that require values between zero and one.

  - Contract rates: LGDs are within the interval [0,1];
  - All other concepts may result in negative LGDs. However, the likelihood for negative LGDs is small.

Table 2 summarises the WG view on these approaches. The criteria are phrased in the positive and evaluation categories may be: ***/agree, **/neutral, and */disagree.

Table 2: Evaluation of approaches

Criteria	**Contract rate**	**WACC**	**ROE**	**Bond return**	**Equilibrium return**
Guiding principles of discount rates	*	***	*	*	***
Simplicity for:					
SMEs	***	**	**	*	*
Large corporates	***	**	**	**	*
Financial institutions	***	**	**	**	*
Credit lines, derivatives and guarantees	**	**	**	*	*
Application to GCD data	**	***	*	*	***
LGD values in [0,1]	***	**	**	**	**

Two approaches have a preference in the Working Group as they meet the guiding principles and can be applied in a data rich environment that is available to GCD members:

- WACC based on the assumption that post-default capital is a reasonable reflection of systematic risk;
- Market equilibrium return based on the assumption that the link between measure for systematic risk and sensitivity to market excess returns is reasonable.

4 Applying different discount rates to workout data

4.1 Using GCD workout data for empirical discount rate studies

Global Credit Data is a not-for-profit initiative to help banks to measure their credit risk, owned by its 50 member banks across Europe, Africa, North America, Asia and Australia. GCD has collected one of the world's largest LGD/EAD databases with a large number of defaulted facility observations totalling over €200 billion in all Basel asset classes.

This GCD study analyses this database and is, to our best knowledge, the first to estimate discount rates from observed resolution information. The use of the GCD database for LGD discount rates has a number of merits:

- Discount rates are based on systematic risk. The large number of defaulted facilities in the GCD database allows for the measurement of systematic risk, i.e., the remaining non-diversifiable risk in a diversified portfolio.
- The data set is sufficiently large to form risk segments such as geographies and industries, which allows for an estimation of segment-specific systematic risk and hence the required risk premiums.
- Most discount rate approaches may be inferred from GCD data. We base the costs of equity on a bank beta of one and the cost of debt on the risk-free rate. The database mainly captures banking book data on non-traded bank instruments. Hence, the market return approach is not included in the empirical part of this study and we refer the reader to Jacobs Jr (2012).

4.1.1 Screening the data and applying data filters

The empirical analysis is based on data provided by GCD in June 2015. For the purpose of this analysis, member banks agreed to a selection of filter rules as shown in Table 3, which shows the number of defaulted loan facilities and borrowers after the application of various filter rules.

29,569 defaulted facilities in relation to 17,193 borrowers remain after the application of these filters. We analyse LGDs on the facility level. A robustness check reveals that there is only a minor difference between facility and borrower based mean LGD per default year, as the majority of borrowers relates to a single facility. Note that it is possible that some banks consolidate multiple defaulted loans by the same borrower into a single facility.

Table 3: Number of defaulted facilities and borrowers after the application of filter rules

Filter rules	Facilities	Borrowers
Raw Dataset	123,577	64,140
Facilities with FC, LC and SME as borrower	103,089	52,240
Resolved facilities	88,966	45,916
Unsecured facilities	34,498	20,395
Defaulted in 2000-2013	32,521	19,294
Various robustness checks[19]	31,806	18,924
Borrower EAD > 10K Euro	29,569	17,193

FC are financial companies, LC are large companies and SME are small- and medium-sized enterprises.

4.1.2 Risk segmentation

In terms of risk segmentation, the following risk segments are being formed in terms of geographical and cultural proximity:

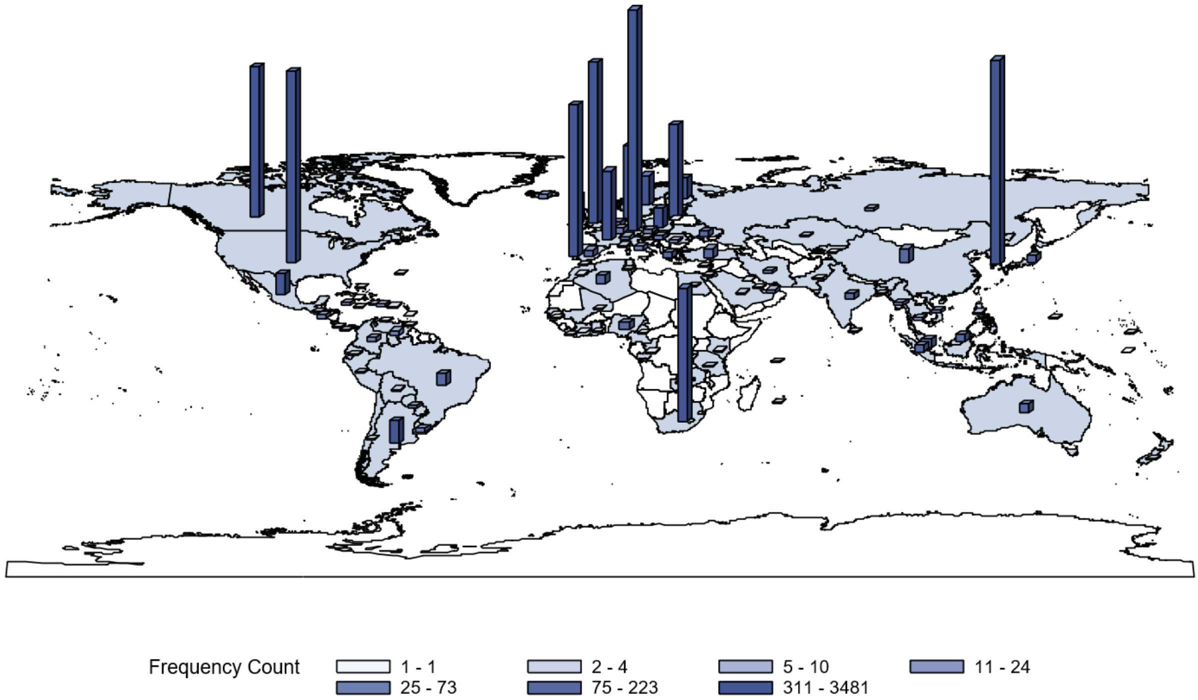
- Great Britain and Ireland;
- Central Europe[20]: Austria, Belgium, France, Germany, Luxemburg, Netherlands and Switzerland;
- Hispania: Portugal and Spain;
- North America: Canada and United States
- Scandinavia: Denmark, Finland, Norway and Sweden;
- South Africa;
- Others: all other countries.

Figure 2 shows the number of defaulted facilities after data filtering by country on a world map.

[19] These checks include: (i) a test to see whether the amount of write off and cash flows is reasonable, (ii) loan reported ‘Resolved’ showing all transactions in excess of Exposure by less than or 10 %, (iii) variable ‘Entity Asset Class’ must be given, (iv) if the Facility Asset Class = SME or Large Corporate, then the Entity Asset Class must equal Corporate, (v) if the Facility Asset Class equals 'Banks & Fin Co', then the Entity Asset Class must equal Banks or 'Non-bank Financial Company’, (vi) Entity Asset Class must equal Banks or 'Non-bank Financial Company' when Primary Industry Code equals 'Finance and Insurance', (vii) Facility Asset Class must be given, (viii) Facility Asset Class must equal 'Banks & Fin Co' when Primary Industry Code equals 'Finance and Insurance', (ix) Loan Status must be given in the History table at least once for each Facility ID.

[20] The working group is aware that Central Europe may be defined differently. We label Central Europe a set of geographies which the WG believes features risk characteristics that are close to each other.

Figure 2: Total number of observations, by country

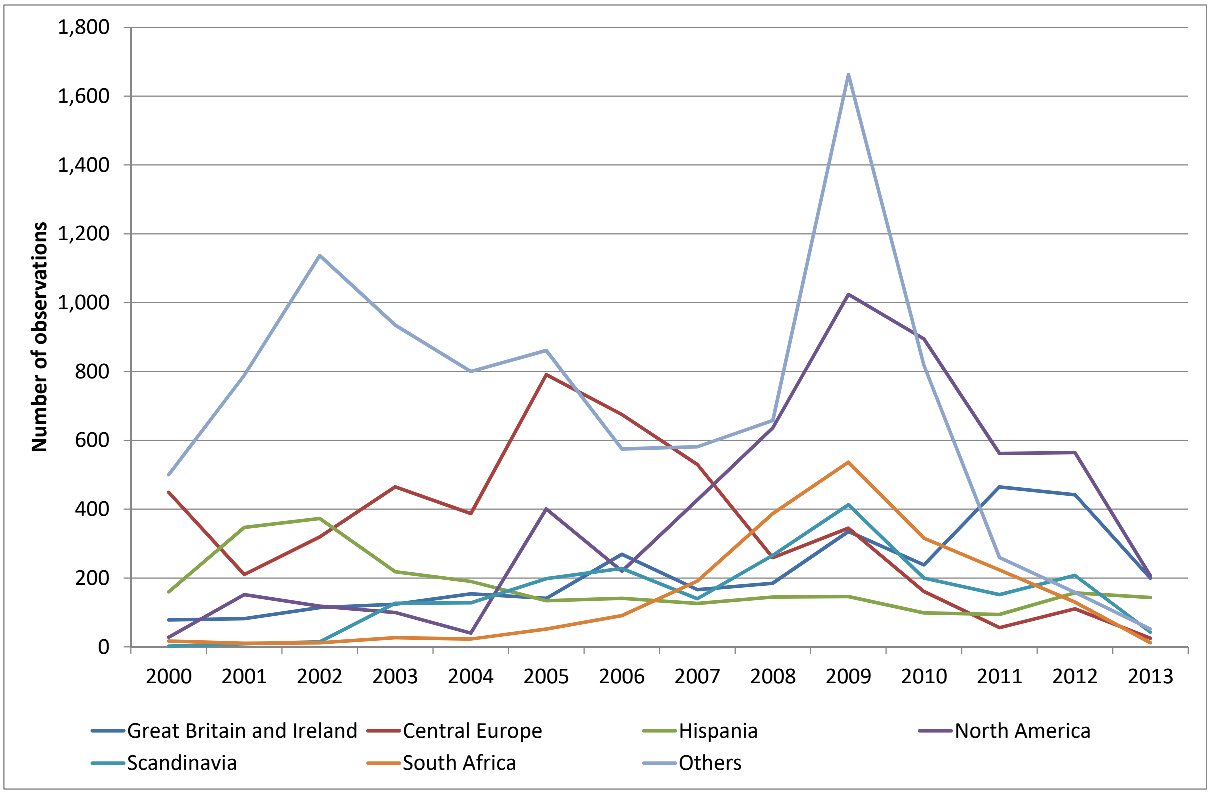


We have chosen the following risk segments in terms of industry segmentation:

- Commerce: Wholesale and Retail Trade;
- Construction;
- Finance: Real Estate and Rental and Leasing, Finance and Insurance;
- Manufacturing;
- Services: Transportation and Storage, Professional, Scientific and Technical Services, Education, Extra-Territorial Services and Organisations, Health and Social Services, Hotels and Restaurants, Other Community, Social and Personal Services, Private Sector Services (Household);
- Others: Agriculture, Communications, Hunting and Forestry, Fishing and Fishing Products, Mining, Public Administration and Defence, Utilities.

The classification by geographic and industry follows prior literature as economic cycles are commonly seen as country- and industry-specific. The reported categories were selected based on a minimum 100 loss events observations per default year. More granular classifications (e.g., individual countries within Scandinavia) were analysed with consistent results. Figure 3 shows the number of observations per risk segment:

Figure 3: Total number of observations by geography risk segment



The reason for this segmentation strategy is that we measure the exposure to systematic risk based on the time variation of loss rates. This requires the assumption that loss rates are not exposed to idiosyncratic risk, for which we require on average a minimum of 100 loss observations per year.

The consequence of defining more granular risk segments[21] would be that the observed loss rate variation is based on a combination of systematic and idiosyncratic risk and the exposure to systematic risk is overestimated.

[21] One example would be to break out and combine the three industries "Agriculture, Hunting and Forestry", "Fishing and Fishing Products" in one category. This would imply a total of 904 loss observations (from 2000 to 2013) with fewer than 50 loss events per year in economic upturns (2001 and 2002) and the remainder in economic downturns (from 2007 to 2009).

4.1.3 Empirical strategy

The GCD data collects a number of information tables. We applied the following databases to set filter rules:[22]

- **Entity:** general information on the borrower or guarantor;
- **Loan:** general information on the facility;
- **History:** dates for five events: origination, one year prior to default, default, post default and resolution.

We have applied the following data set to obtain the timing of origination, default and resolution:

- **History:** dates for five events (see above).

We have applied the following data set to obtain the risk-free GCD discount rate:

- **Loan:** general information on the facility.

We have applied the following data set to obtain the gross resolution cash flows:

- **Transaction:** transaction type, date and transaction (amount) cash flows.

In the data analysis, we follow a number of consecutive steps:

1. Application of filter rules;
2. Risk segmentation: all facilities are grouped into geographic and industry classes;
3. Computation of nominal LGD based on the LGD 2 definition and a zero discount rate (LGD based on risk-free rate is provided in the GCD database);[23]
4. Correction of risk-free and nominal LGD based on risk-free rate and zero discount rate for time to resolution bias;
5. Estimation of ENLGD as the long run average over ONLGD and ELGD as the long run average over OLGD;
6. Merging of macro-economic risk drivers: real GDP growth and lagged mean log recoveries (from default to resolution time);
7. Computation of systematic risk measures as dispersion of a frailty effect, which models the deviation of the log recoveries from the long run average based on observed log recoveries;
8. Computation of contract rate (Contract) and pre-default expected return (Contract 2);
9. Computation of cost of equity (ROE) and the weighted average cost of capital (WACC);
10. Computation of equilibrium returns (Equilibrium);

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[22] We did not apply the remaining GCD data sets, which are Financial: entity, financial (sales, assets and debt), Guarantor: guarantor credit risk information, and Collateral: general information on collateral.

[23] The LGD 2 definition includes the sum of present values of all cash flows, excluding principal advances and financial guarantees, except write-offs and Interest Accruals, which are not cash flows. Please refer to Global Credit Data (2015).

11. Computation of observed LGDs based on the discount rates;
12. Correction of observed LGD for time to resolution bias;
13. Production of statistics for discount rate analysis (Section 4.2) and LGD analysis (Section 4.3).

4.2 Empirical discount rate analysis

We analyse the risk-free rate (GCD), contract rate (Contract), adjusted contract rate (Contract 2), weighted average cost of capital (WACC), return on equity (ROE), and equilibrium return (Equilibrium).

4.2.1 Assumptions

We follow the computations of Section 3.3 and make the following assumptions:

- Risk-free rate (GCD): based on the three-month EURIBOR rate and the short-term interest rates of the respective country. Note that EURIBOR is an interbank lending rate for unsecured funds and includes an average bank credit spread.
- Contract rates (Contract): contract rates are observable for approximately 13% of all observations at facility origination. The majority (74%) of facilities that have pricing information are priced with a floating rate. We extract the spread in excess of the base rate for floating rate loans and compute the contract rate as the sum of the spread and the risk-free rate at default. We replace missing values by median contract rates (for both the contract rate and the pre-default expected return) by geography and default year.
- Adjusted contract rate (Contract 2): we consider the contract rate and the estimated pre-default expected return. The discount rate is calculated following Equation (15). The mean discount rate is 3.72% and an alternative computation using Equation (12) has resulted in similar discount rates: 3.46% if ELGD is estimated based on the risk-free rate and 3.84% if ENLGD is used to avoid the circular reference. The economic impact on LGDs is comparable.
- Weighted average cost of capital (WACC): we assume for the Downturn LGD (which is an input to the WACC concept) the maximum average LGD over default years and by geography. This may be similar to conditioning on historic adverse macroeconomic states. Banks may apply alternate downturn LGD models. The discount rate is calculated following Equation (20). We avoid circular references by estimating ELGD as the mean of the observed LGD for a given risk segment based on the risk-free rate. Robustness checks using ENLGD give comparable results.

- Return on equity (ROE): a bank beta measure of one is assumed, the discount rate is calculated following Equation (22).
- Equilibrium return (Equilibrium): the discount rate is calculated following Equation (25). The beta measure is based on Equation (25) and Equation (26). Table 4 shows the resulting parameter estimates for the point-in-time regression for log recoveries applying segmentation by geography and a segmentation by industry. Int is the intercept estimate for the regression model.

Table 4: Parameter estimates for a PIT regression model for log recoveries with frailty effects

Segment	Int.	GDP	Avg. log RR	gamma	delta	AC	beta
Gr Britain and Ireland	-0.4319	-0.0518	0.3909	0.3811	1.1413	0.1003	0.5631
Central Europe	-0.3820	-0.0005	0.2437	0.1553	1.1084	0.0193	0.2467
Hispania	-0.2360	0.0327	0.2828	0.2389	0.9493	0.0596	0.4339
North America	-0.3548	0.0182	0.2389	0.2224	0.6593	0.1022	0.5682
Scandinavia	-0.5347	-0.0068	-0.0594	0.1619	0.9092	0.0307	0.3116
South Africa	-0.1793	-0.0004	0.7120	0.2099	0.9876	0.0432	0.3696
Others	-0.5961	0.0079	0.0475	0.1647	1.1726	0.0193	0.2473
Commerce	-0.2194	-0.0014	0.6112	0.1622	1.0413	0.0237	0.2737
Construction	-0.2215	-0.0073	0.3605	0.1604	0.8721	0.0327	0.3216
Finance	-0.3312	-0.0017	0.3243	0.1721	0.9375	0.0326	0.3210
Manufacturing	-0.2552	0.0027	0.4623	0.0942	0.9966	0.0089	0.1673
Services	-0.2406	-0.0030	0.5011	0.1580	0.9981	0.0245	0.2780
Others	-0.2138	-0.0032	0.7037	0.1131	1.1904	0.0089	0.1681

Note: This table provides regression results in accordance with Equation (26).

In the empirical analysis we have applied the reference values from Maclachlan (2004) $\sigma_j = 0.32$ and $\sigma_M = 0.18$ in conjunction with the AC estimates from Table 4 to estimate betas and discount rates and compute the LGDs given these discount rates. In this model, we include two systematic variables: real GDP growth and mean log recovery, which are lagged by one period. Both variables have been shown to be powerful systematic control variables for default risk (see e.g., Lee et al., 2016). We avoid circular references by computing ORR as 1-OLGD based on the risk-free rate. Robustness checks using ONLGD resulted in comparable results.

All discount rates are based on the combination of risk-free rate and a spread:

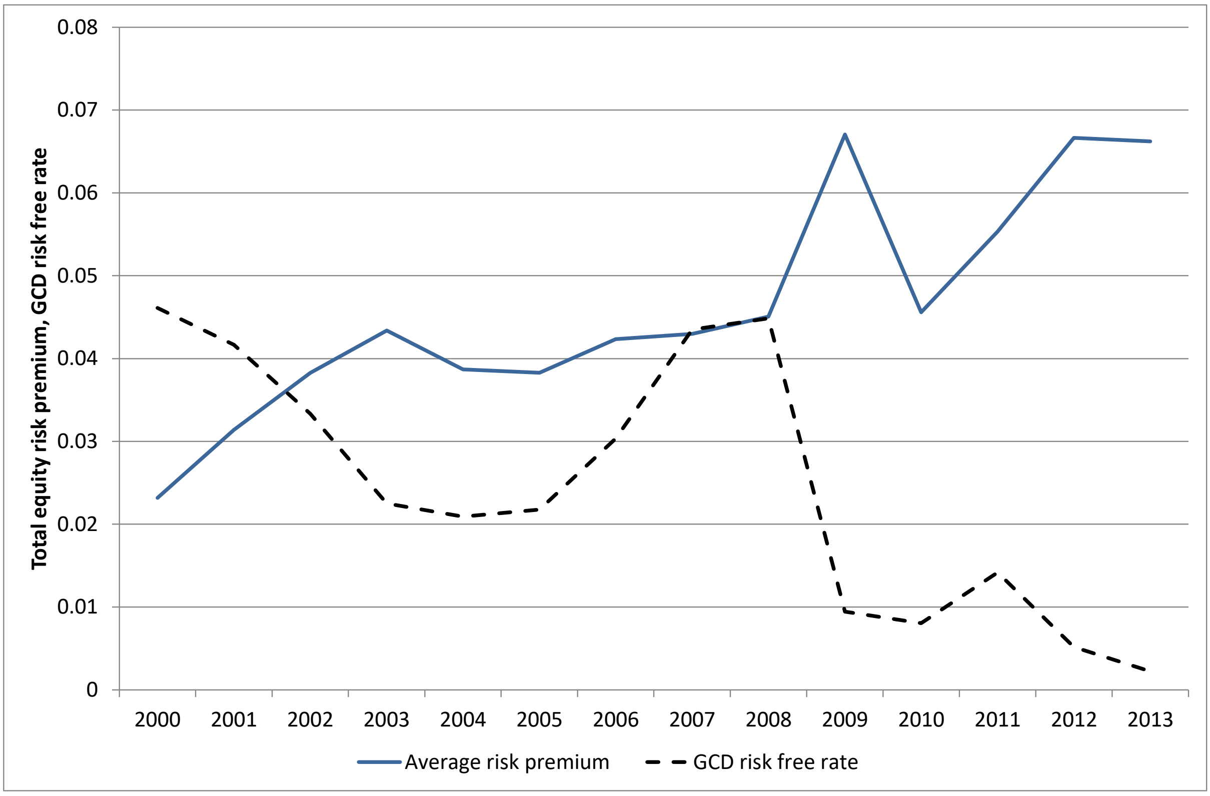
- GCD: risk-free rate at default;
- Contract rate: risk-free at default plus credit spread at origination;
- Contract 2: risk-free rate at default plus credit spread at origination less expected loss;
- ROE: risk-free rate at default plus equity risk premium at default times beta (assumed to be equal to unity);
- WACC: capital ratio times ROE at default plus debt ratio times risk-free rate at default;

- Equilibrium: risk-free rate at default plus equity risk premium at default times beta based on the systematic variation of mean LGDs over time (see next section).

The spread for ROE, WACC and Equilibrium approaches is based on the equity risk premium.

Whilst the CAPM theory does not provide for risk premiums outside the co-movement with the market return, the literature has pointed out that more volatile financial markets attract a greater risk premium, which we include into our analysis (see Damodaran, 2015). Other factors for risk premiums (e.g., premiums for size where smaller firms attract a higher risk premium) may exist but we do not include this aspect into our analysis as our knowledge of the borrowers underlying the defaulted facilities is limited.

Figure 4: Average total equity risk premium and GCD risk-free rate



The risk-free rate is based on the GCD discount rate and the implied equity risk premiums are from Damodaran (2015), who publishes annual country risk premiums based on an average risk premium (RP) and the time-varying risk premium implied by future dividends (IRP) for the US (see http://pages.stern.nyu.edu/~adamodar). We computed an implied country risk premium on a country level by:

$$IRP_{kt} = RP_{kt} - RP_{US,t} + IRP_{US,t} \tag{29}$$

For example, for Canada in 2000 we observed the following values: $RP_{Canada,t} = 0.0611$, $RP_{US,t} = 0.0551$, and $IRP_{US,t}$=0.0205. We compute the IRP for Canada as follows:

$$0.0265 = 0.0611 - 0.0551 + 0.0205$$

We define the index for a country as k (which is different from the broader risk segmentation j used before). The assumption behind the computation is that the US market (which is the largest in size) provides a base equity risk premium for national risk premiums. Figure 4 shows the average total equity risk premium as the sum of a global market risk premium and a country risk premium (coloured solid lines) and the risk-free GDC discount rate (black dashed line). Equity risk premiums and risk-free rates offset and may mitigate the time variation of discount rates during the period 2000 to 2013.

4.2.2 Euro risk-free rates

In this section, we use Euro as a pivot currency. Transaction amounts and amounts at default are converted and expressed in Euro. The LGDs are then calculated using those converted amounts. Consistently with the Euro denominated cash flows, the EURIBOR rate is used as a risk-free rate for all facilities regardless of geography.

Figure 5: Median discount rates over time, observations with recorded contract rate

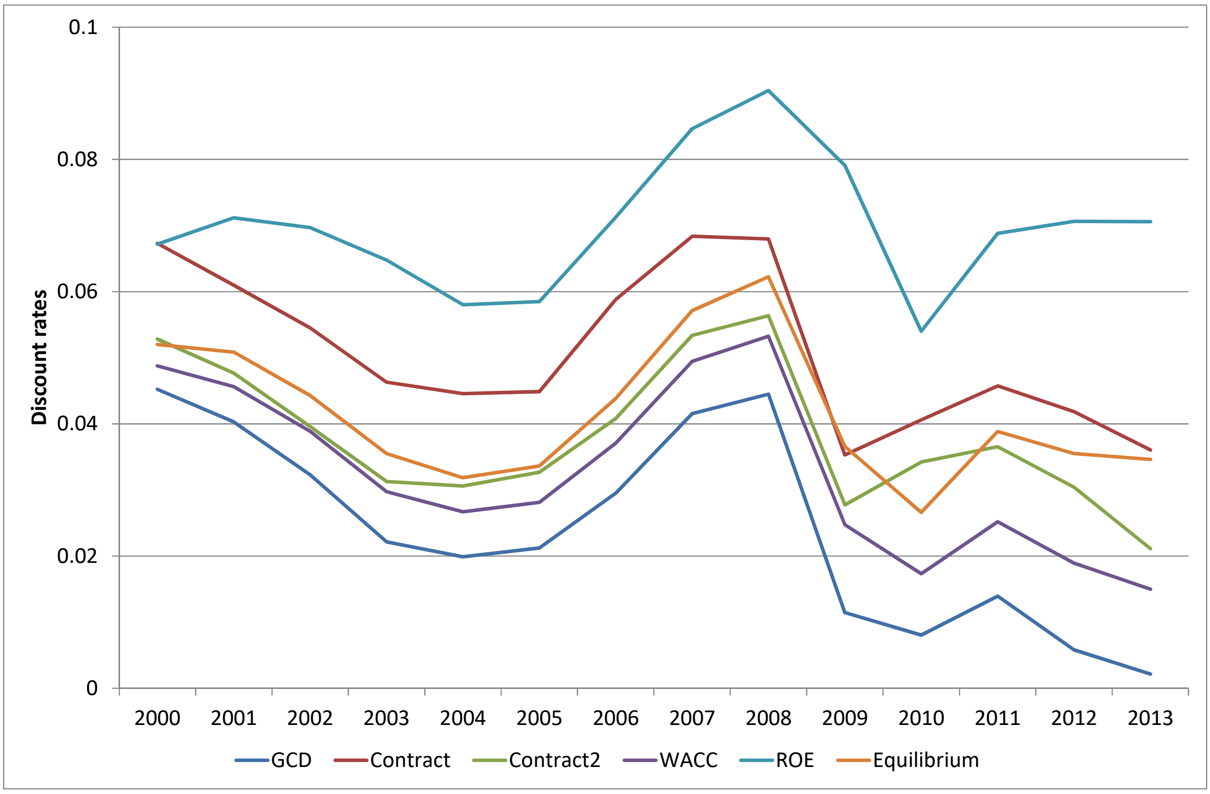


Figure 5 shows the resulting discount rates. The lowest discount rate is the risk-free rate (GCD) and the highest discount rate is the ROE. Contract rates and market-implied discount rates are consistent, which is a reflection of the integration of lending and capital markets. Table 5 shows the moments for the main discount rates.

Table 5: Descriptive statistics discount rates

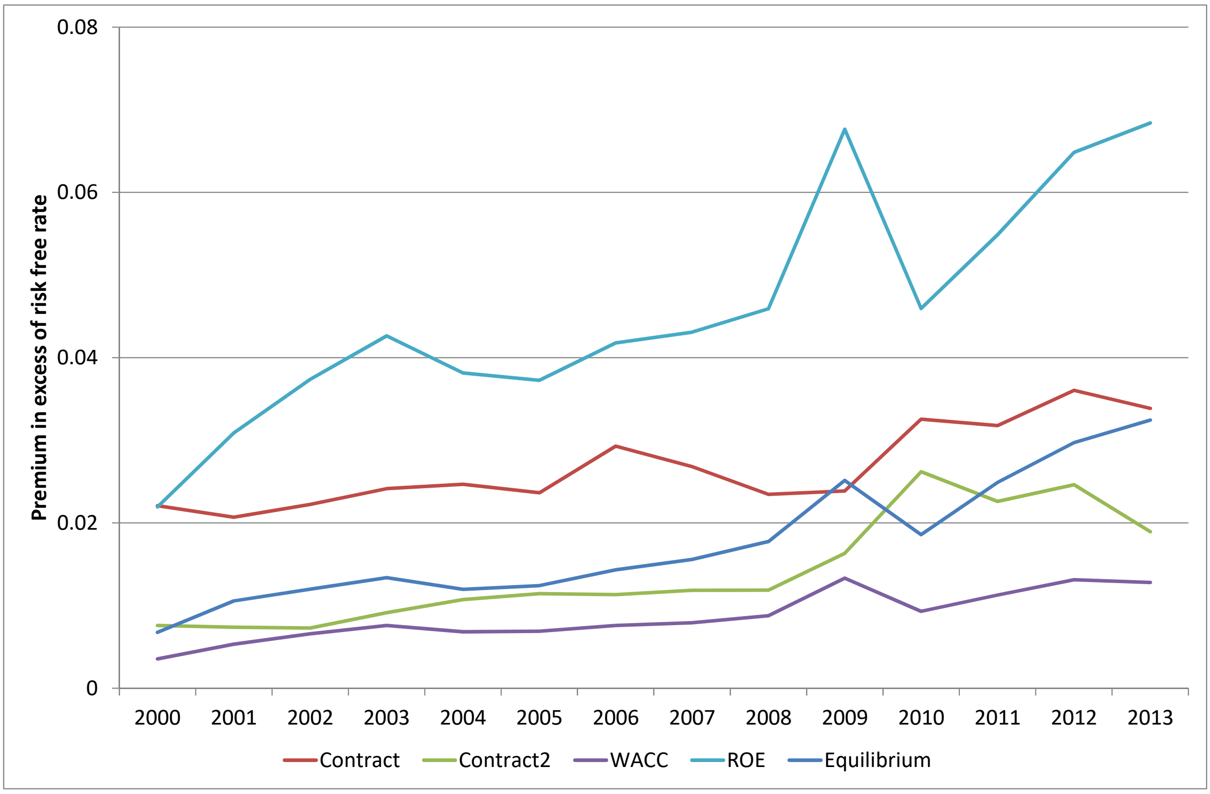
Measure	GCD	Contract	Contract2	WACC	ROE	Equilibrium
N	29,569	2,560	2,560	29,569	29,569	29,569
Min	0.0000	0.0000	-0.0275	0.0027	0.0205	0.0051
Max	0.0539	0.2762	0.1946	0.0647	0.1168	0.0782
Mean	0.0236#	0.0491	0.0372	0.0326	0.0707	0.0413
StdDev	0.0147	0.0229	0.0214	0.0131	0.0136	0.0136

For the avoidance of doubt, in this table, 0.0236 is to be read as 2.36%.

Note, Contract 2 is computed following Equation (15) which is corrected for the expected loss. A negative value is unlikely but possible if expected losses exceed the contractual rate. In most instances, this can be attributed to low contractual rates.

Figure 6 shows the mean premiums in excess of EURIBOR which increase during the reference period.

Figure 6: Mean premiums in excess of EURIBOR over time, joint cross-section, observations with recorded contract rate



All risk-adjusted approaches are directly (equilibrium approach) or indirectly (other approaches) based on market prices for the time value of money (risk-free rate), systematic risk (market risk premium) and exposure to systematic risk. This implies that loan exposures with a higher risk-free rate, a higher market price for systematic risk or a higher exposure to systematic risk result in higher discount rates. High levels of default risk for a borrower, an industry, or a country do not necessarily imply a high discount rate as a high level of idiosyncratic risk may not be causal for a high level of systematic risk and a high market price.

4.2.3 Currency-specific risk-free rates

The previous analysis has used Euro as a pivot currency. The following figure shows the short-term interest rates which co-move for GCD countries. This includes South Africa (currency ZAR, top line) which has a higher level.

Figure 7: Mean discount rates by currency

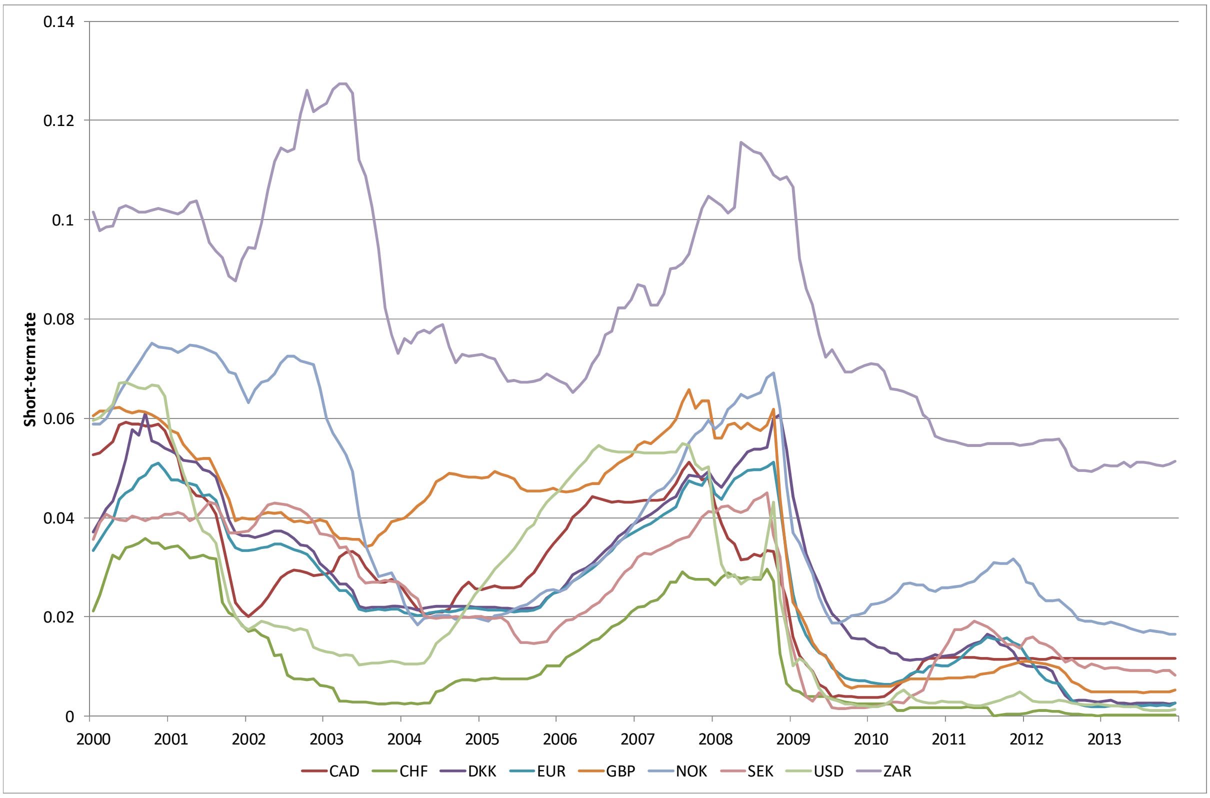
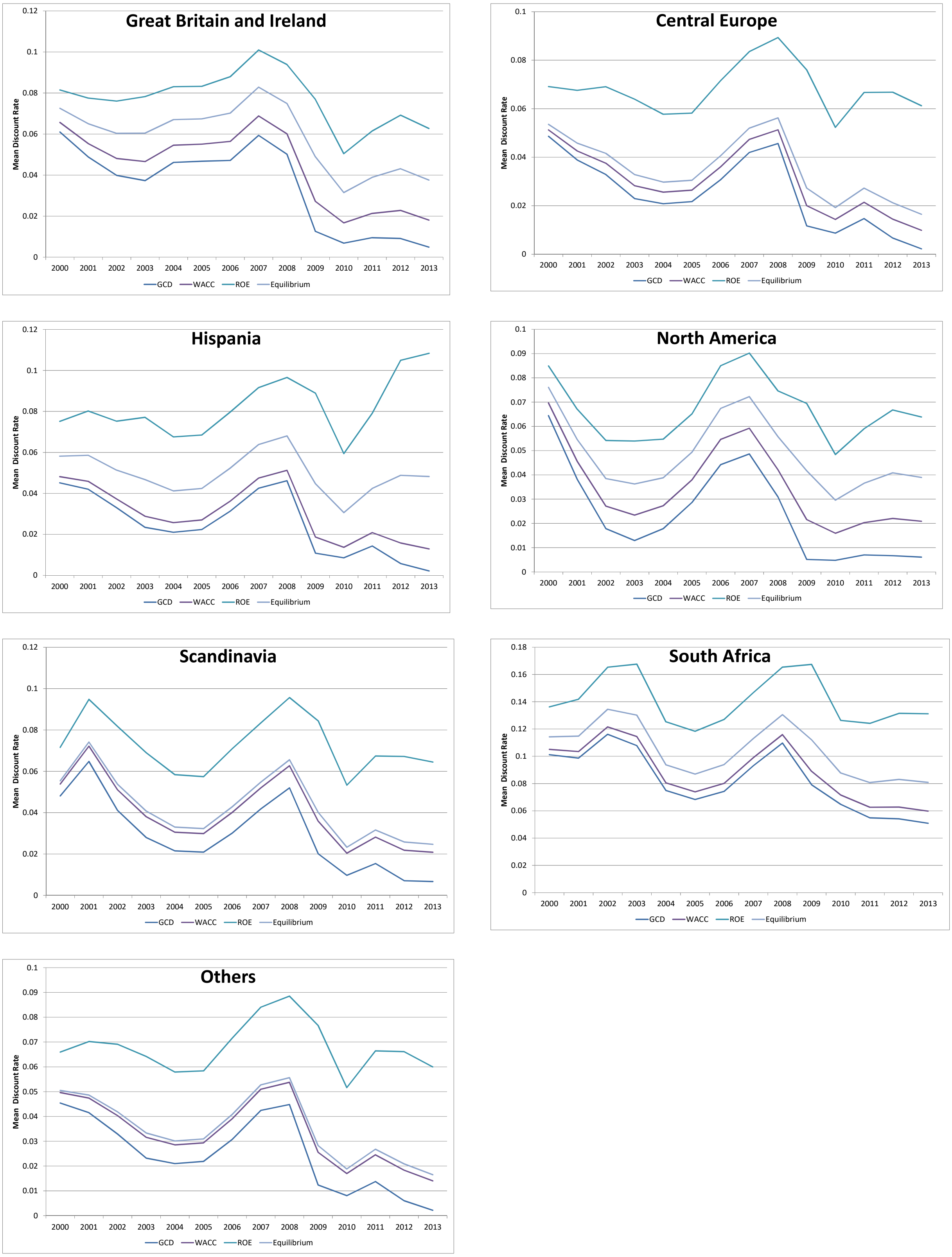


Figure 8 shows the mean discount rates by geography. We do not report the contract rate as the number of observations is insufficient. Differences in mean currency specific risk-free rates are co-moving over the countries analysed. This shows the integration of the various economies. Differences may be attributed to the sovereign credit spread. Switzerland has the lowest rates and South Africa the highest rates of the countries analysed.

Figure 8: Mean discount rates by geography



The EURIBOR rate is lowest, followed by the WACC rate, Contract2 rate, Equilibrium rate, Contract rate and ROE based on an asset beta of one. The Contract rate is relatively high as it includes a compensation for the expected loss. ROE is highest as a beta of one is assumed, whilst most discount rates suggest an implied level of systematic risk, which is equivalent to a beta of less than one.

4.3 Empirical LGD analysis

4.3.1 Euro risk-free rates

In this section we compute and compare the LGDs that result from the various discount rate concepts without further corrections for resolution bias. Resolution bias will be scrutinised in the next section.

Figure 9: Mean LGD (in %), without correction for resolution time bias

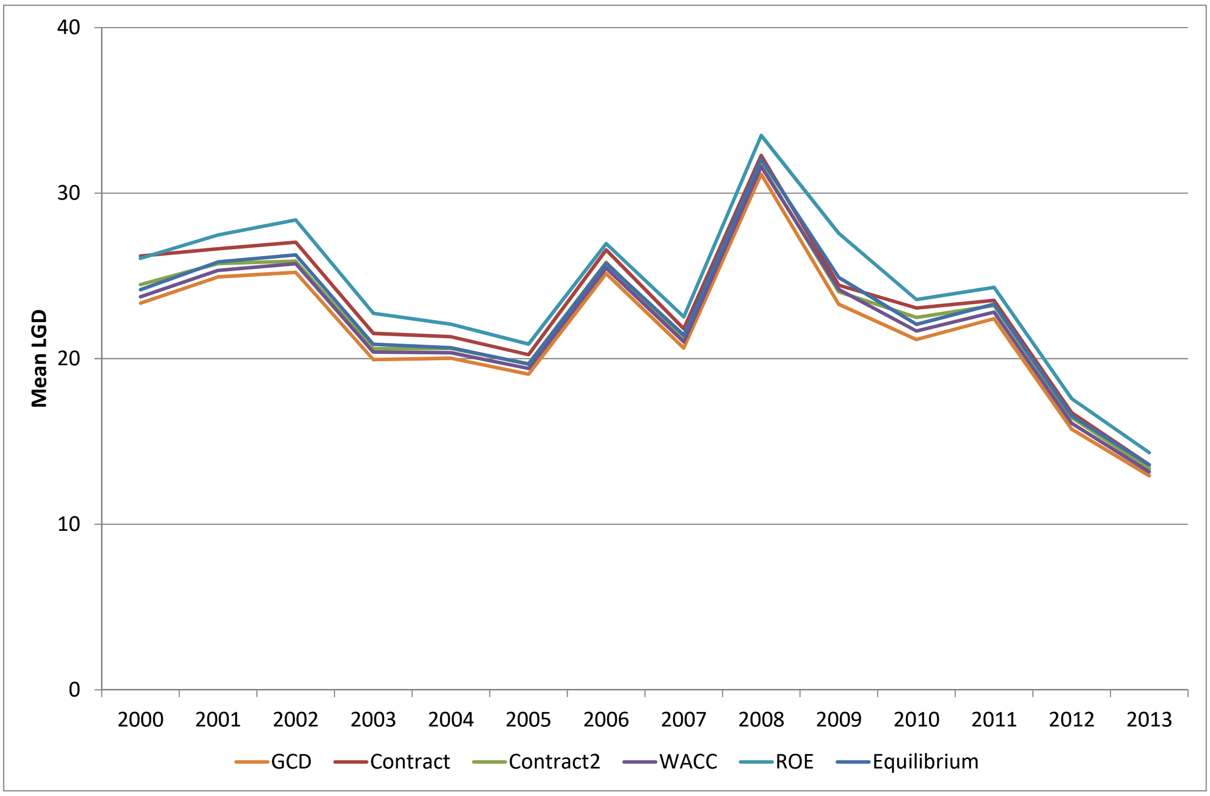


Figure 9 shows the mean LGD without correction for resolution time bias. The discounting of recovery cash flows implies that a lower discount rate results in a lower LGD. The variation of LGDs given different discount rate concepts is limited and decreases with time to resolution in recent years. Table 6 shows the moments for the mean LGD, without correction for resolution time bias.

Table 6: Descriptive statistics LGDs (in %), without correction for resolution time bias

Measure	GCD	Contract	Contract2	WACC	ROE	Equilibrium
N	29,569	29,569	29,569	29,569	29,569	29,569
MIN	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MAX	150.0000	150.0000	150.0000	150.0000	150.0000	150.0000
MEAN	22.4122#	23.8168	23.1600	22.8887	24.9517	23.3500
STD	35.9819	35.6135	35.7517	35.8369	35.3605	35.7172

For the avoidance of doubt, in this table, 22.4122 is to be read as 22.4122%.

4.3.2 Correction for time to resolution bias

The data sample is subject to a resolution bias as unresolved LGDs with a higher time to resolution are not analysed. Betz et al. (2017) document for the same data set a positive correlation between time to resolution and LGDs for corporate loans. Do et al. (2018) document the same finding for mortgage loans. To correct for this bias, we adjust the mean LGDs ($LGD_{t,adjusted}$) as follows:

$$LGD_{t,adjusted} = CR_t * LGD_{t,resolved} + (1 - CR_t) * LGD_{t,unresolved} \tag{30}$$

Table 7: Resolved LGD (in %), completion rate, estimated unresolved LGD and adjusted LGD.

Default Year	Obs.	Resolved LGD	CR	Unresolved LGD	Adjusted LGD
2000	1,234	23.3581	0.9724	70.6098	24.6613
2001	1,599	24.9387	0.9423	70.6098	27.5762
2002	2,089	25.2126	0.9368	70.6098	28.0830
2003	1,996	19.9498	0.9541	70.6098	22.2746
2004	1,722	20.0275	0.9498	70.6098	22.5664
2005	2,578	19.0705	0.9457	60.5835	21.3243
2006	2,199	25.1420	0.9524	47.5273	26.2085
2007	2,161	20.6456	0.9134	44.3458	22.6991
2008	2,536	31.1533	0.8493	42.3420	32.8395
2009	4,463	23.2844	0.8780	41.8823	25.5529
2010	2,728	21.1609	0.8289	40.1759	24.4139
2011	1,812	22.4168	0.7451	36.9915	26.1324
2012	1,771	15.7451	0.6856	35.1343	21.8404
2013	681	12.9344	0.6329	31.9048	19.8985

CR_t is the completion rate, which is the fraction of defaulted loans that have been resolved for a given default year. $LGD_{t,resolved}$ is the mean resolved and hence observed LGD. We estimate the mean of unresolved (and hence unobserved) LGDs for a given default year by the mean over earlier resolved LGDs with a time to resolution (TTR) greater than the observed LGDs for a given default year:

$$LGD_{t,unresolved} = \frac{1}{\sum_{i=1}^{N} I(TTR_{it,resolved} \geq 2015.5 - t)} \sum_{i=1|TTR_{it,resolved} \geq 2015.5-t}^{N} LGD_{it,resolved} \tag{31}$$

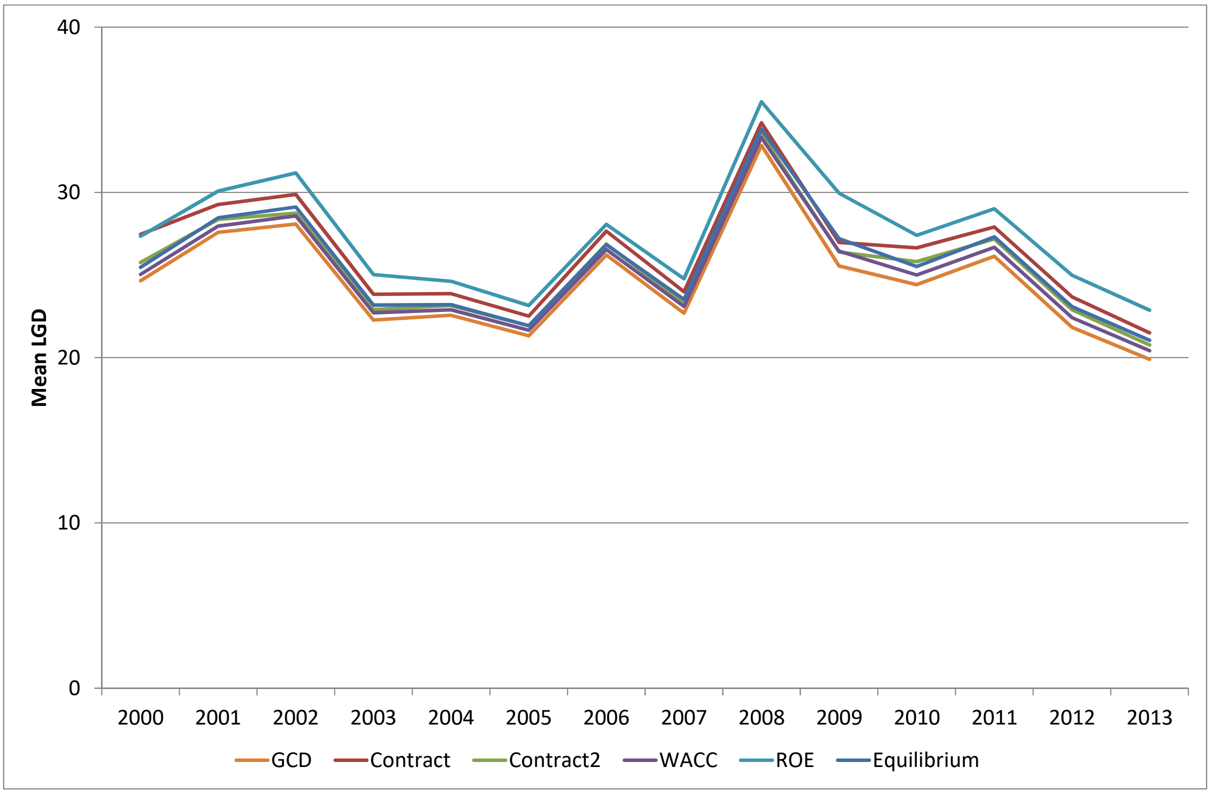
Table 7 shows by default year the resolved LGD before correction, completion rates, estimated unresolved LGD and adjusted LGD after correction (LGDs are based on the risk-free rate).[24]

For example, given $LGD_{2013,resolved}$*=12.93, CR=63%, t=2013, end of observation period is 2015.5 (i.e., mid-2015). The mean LGD of unresolved LGD (i.e., defaulted loans with a time to resolution of over 2.5 years) is 31.90. The adjusted mean LGD is computed as follows:* $LGD_{2013,adjusted} = 12.93 * 0.63 + 31.90 * (1 - 0.63) = 19.90$.

[24] For the avoidance of doubt, in this table, 23.3581 is to be read as 23.3581%.

Figure 10 shows the mean LGD with correction for resolution time bias:

Figure 10: Mean LGD (in %), with correction for resolution time bias



The mean LGDs are higher than in Figure 9 over all default years as the decreasing completion rate (from 2000 to 2013) is offset by a decreasing implied LGD (from 2000 to 2013) which is higher than the observed LGD in all instances.

The choice of discount rates has a low to moderate impact, which is a function of the following two aspects:

- Average resolution time which is decreasing from 3.16 in 2000 to 0.51 in (2013)
- Average equity risk premium: the price for systematic risk is increasing from 2.19 in 2000 to 6.84% in 2013

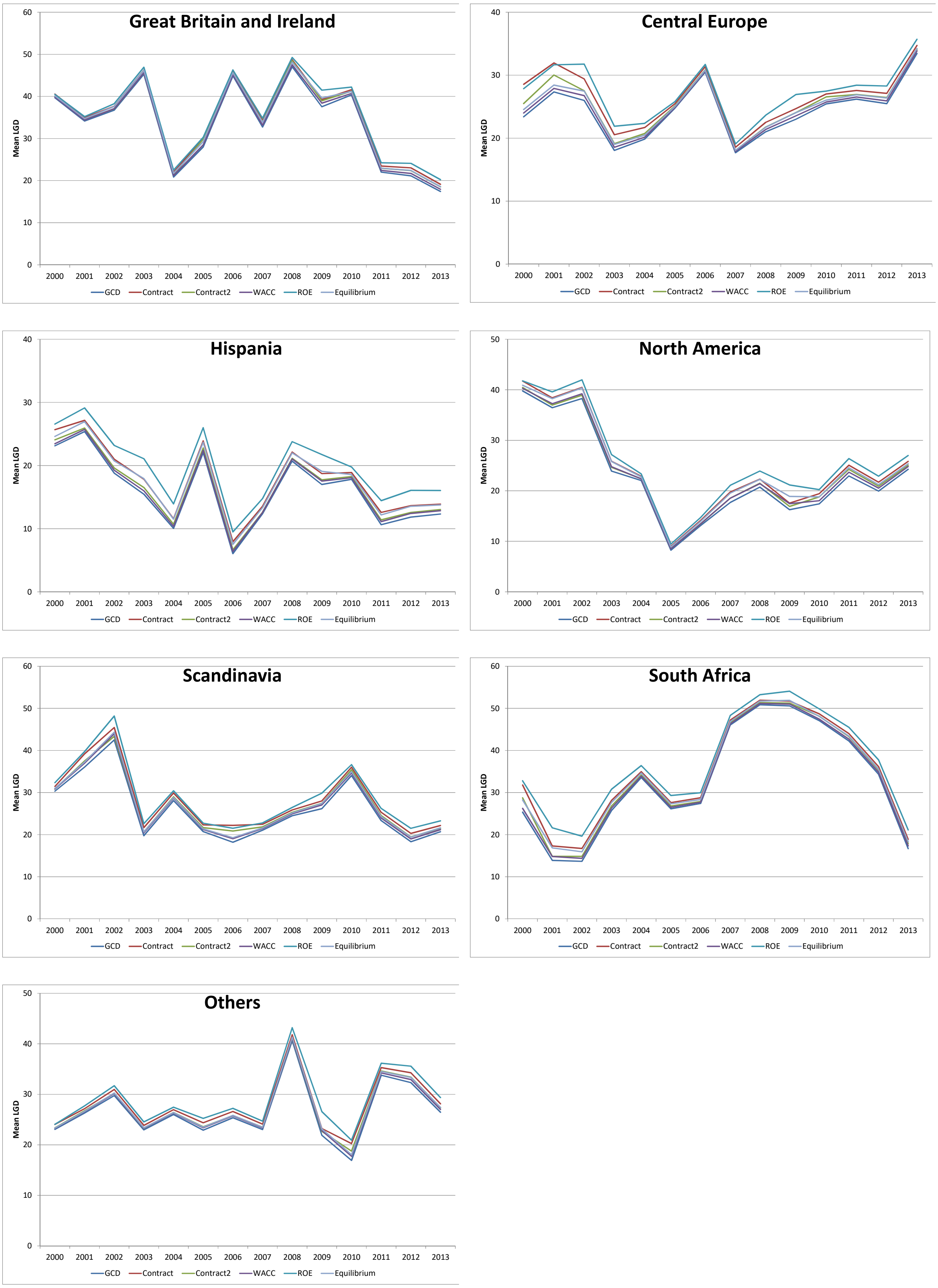
These findings are in line with a paper by Gibilaro and Mattarocci (2011) that analyses the impact of the risk-free rate, contract rate and two equilibrium models (based on the beta between average recoveries and GDP as well as average recoveries and a defaulted bond index). The average LGDs vary between 0.5051 (using a risk-free discount rate) and 0.5327 (using the equilibrium model based on the defaulted bond index). Table 8 shows the descriptive statistics for the main discount rates for all years, the start year (2000) and the end year (2013).

Table 8: Descriptive statistics LGDs (in %) after adjustment for resolution bias

All years							
Measure	**GCD**	**Contract**	**Contract2**	**WACC**	**ROE**	**Equil**	**Equil2**
N	29,569	29,569	29,569	29,569	29,569	29,569	29,569
MIN	1.9475	1.9848	1.9582	1.9524	2.0025	1.9432	1.9648
MAX	147.8104	147.8477	147.8211	147.8153	147.8654	147.8061	147.8277
MEAN	25.0829[#]	26.6813	25.9287	25.5887	27.9014	25.3255	26.1114
STD	31.7808	31.4584	31.5755	31.6553	31.2335	31.7091	31.5493
2000							
Measure	**GCD**	**Contract**	**Contract2**	**WACC**	**ROE**	**Equil**	**Equil2**
N	1,234	1,234	1,234	1,234	1,234	1,234	1,234
MIN	1.9475	1.9848	1.9582	1.9524	2.0025	1.9432	1.9648
MAX	147.8104	147.8477	147.8211	147.8153	147.8654	147.8061	147.8277
MEAN	24.6613	27.3862	25.7986	25.0397	27.3447	24.5213	25.4740
STD	30.5928	30.4993	30.4002	30.5786	30.6137	30.5729	30.5627
2013							
Measure	**GCD**	**Contract**	**Contract2**	**WACC**	**ROE**	**Equil**	**Equil2**
N	681	681	681	681	681	681	681
MIN	11.7122	12.8886	12.3366	12.0870	13.7965	11.8924	12.4638
MAX	77.3427	78.5057	77.9596	77.7118	79.4042	77.5225	78.0872
MEAN	19.8985	21.5108	20.8231	20.4184	22.8705	20.1990	21.0495
STD	19.9879	19.8625	19.9010	19.9433	19.7503	19.9484	19.8662

For the avoidance of doubt, in this table, 25.0829 is to be read as 25.0829%.

Figure 11: Mean LGD (in %), with correction for resolution time bias, by geography, EUR discount rate



Mean LGDs differ in terms of levels and volatilities for geographies (Figure 11) and industries (Figure 12). North America demonstrates the greatest variability with some linkage to the economic cycle: mean

LGDs are highest during the 2001/2002 economic downturn and lowest prior to the GFC in 2005.

The mean LGDs for industries are averages over countries and are more aligned in terms of fluctuations but differ in terms of LGD levels. The difference in mean LGDs due to discount rates follows our previous discussions for the whole data.

Figure 12: Mean LGD (in %), with correction for resolution time bias, by industry, EUR discount rate

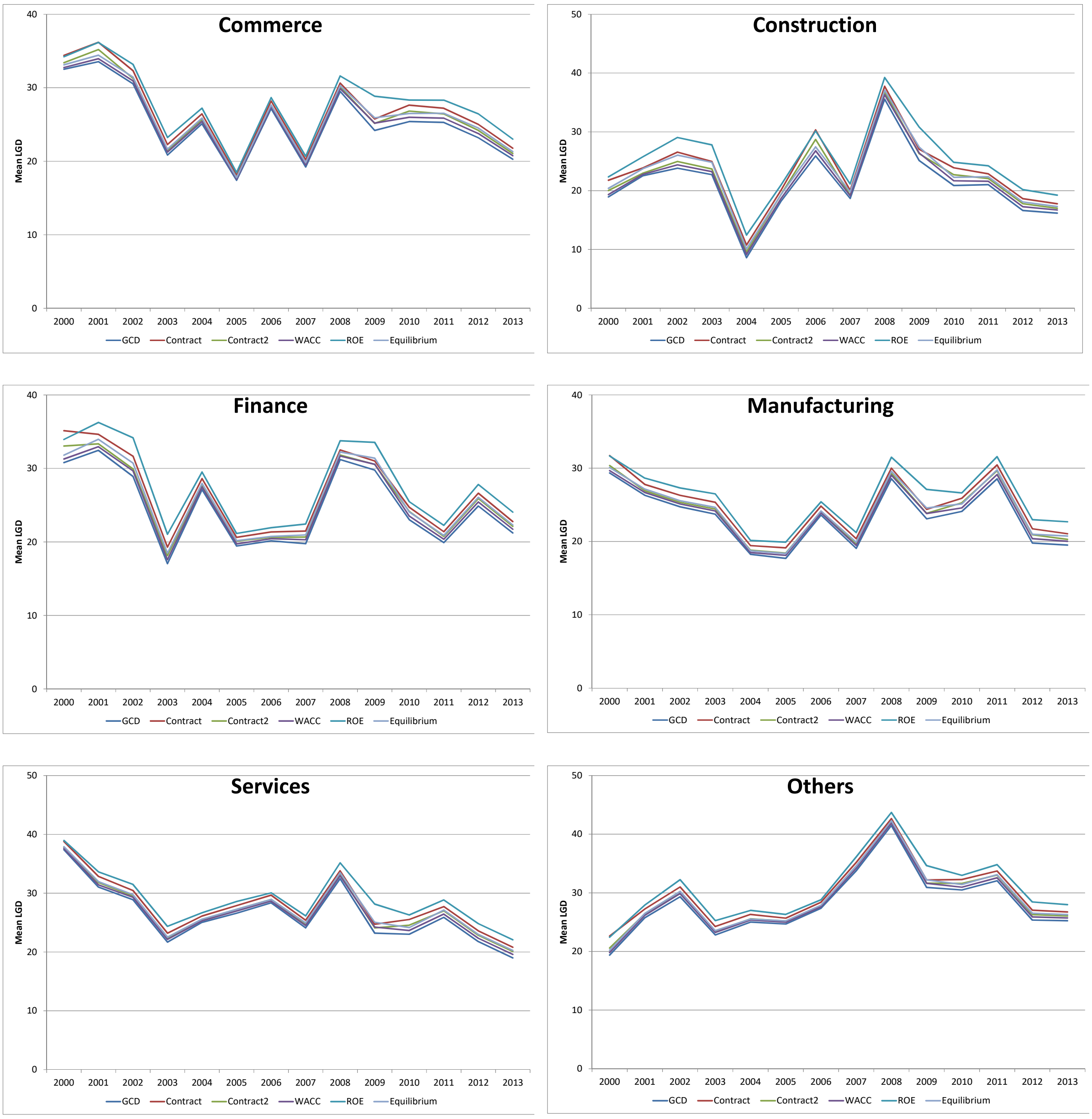


Figure 11 shows the mean LGDs based on the various discount rates for geographies after the correction for time to resolution bias. Furthermore, we provide the mean LGDs based on the various discount rates for industries after the correction for time to resolution bias in Figure 12.

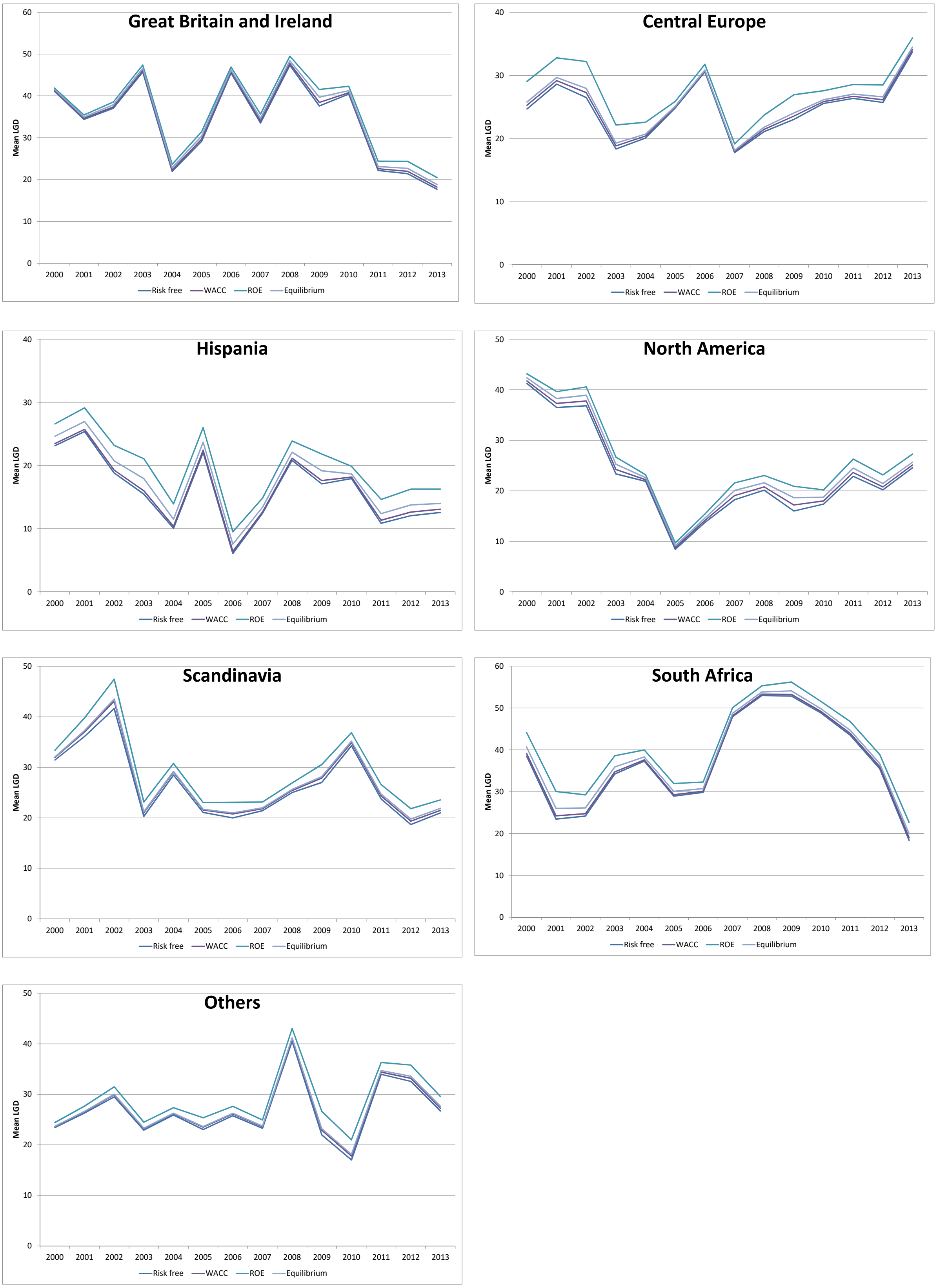
The mean LGDs for industries are averages over countries and are more aligned in terms of fluctuations, but differ in terms of LGD levels. The difference in mean LGDs due to discount rates follows our previous discussions for the whole data.

4.3.3 Currency-specific risk-free rates

Figure 13 shows the mean LGDs based on the various discount rates for geographies, using the original cash flows and the currency-specific short term interest rate reported by the OECD as the risk-free rate.

Contract rates are excluded from this analysis as they are independent from the choice of the risk-free rate. The resulting LGD ranges are similar to the ones reported in Figure 11. Minor differences are visible for South Africa (in particular during the first years) as the country has higher than average risk-free rates translating into higher mean LGDs.

Figure 13: Mean LGD (in %), with correction for resolution time bias, by geography, local short-term rate



5 Conclusion

This paper analyses five LGD discount rate concepts based on four guiding principles (opportunity costs of comparable instruments, risk-free rate and premium for systematic risk at default, exclusion of premiums for realised risk), simplicity, data availability and avoidance of negative LGDs. Furthermore, two methodological advancements were made: the WACC approach was extended for loan-level equity ratios and debt ratios and a correction technique was presented to correct for the resolution bias due to the correlations between LGDs and resolution periods and the right censoring of data.

The paper identifies WACC and market equilibrium return as preferred discount rate concepts as these are in line with the guiding principles and require only limited additional effort relative to the readily available contract rate. Other approaches have some key disadvantages. The contract rate was not the preferred option, as it is based on the origination time and violates the principles that LGDs should relate to the time at default. Further the contract rates include the expected loss. The ROE approach does not measure the systematic risk. Bond returns are not available for general credit risk exposures of commercial banks.

The variation of empirical LGDs is moderate for the various discount rate approaches, as changes in risk-free rates are in part offset by changes in market risk premiums next to limited differences in discount rates and resolution periods and hence, durations.

There is scope for further research work. First, the discipline would benefit from a more granular understanding of the systematic risk of recovery cash flows. Systematic risk measures are challenging to compute. In our case, we used geographic and industry clusters. Further research on systematic risk on other dimensions such as unconditional idiosyncratic risk (e.g., credit score, LTV bands) are needed. Such measures include asset correlation or variation coefficients (e.g., ratio of the standard deviation of the average LGD to the mean LGD over time). New methodologies may be developed to estimate measures for single borrower or loan exposures.

Second, discount rates are also important for other credit risk applications. For example, in IFRS 9 and CECL lifetime expected loss modelling, discount rates may be required. At present, accounting boards require the effective (contract) rate but future research may scrutinise the assumptions and start a more general discussion beyond existing regulations.

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